to the feeding lines network. Moreover, this transition can be used in the Butler matrix to avoid the unavailable cross lines.

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A SURFACE-BASED INTEGRAL-EQUATION FORMULATION FOR COUPLED ELECTROMAGNETIC AND CIRCUIT SIMULATION

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ABSTRACT: *A surface-based integral-equation formulation for coupled electromagnetic and circuit simulation is presented. The approach is sufficiently general to model arbitrarily shaped structures and high-frequency skin effects. The formulation is implemented in both an equivalent circuit form for spice compatibility, and in a more general form as a coupled-matrix system outside spice. The overall approach can be interpreted as either a modified surface-only partial element equivalent circuit approach, or as a circuit-coupled version of the surface-based method of moments.* © 2002 Wiley Periodicals, Inc. Microwave Opt Technol Lett 34: 103–106, 2002; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10386

Key words: *PEEC; method of moments; coupled simulation; integral equations*

1. INTRODUCTION

The simulation of electromagnetic (EM) effects in high-speed chips, packages, and boards is essential for complete electrical characterization. However, it is impractical to analyze the entire structure under test as a large EM problem; this is especially true in iterative design practice. Moreover, the introduction of circuit elements makes the overall problem an inherently coupled and hierarchical one. The partial element equivalent circuit (PEEC) approach [1,2] has been developed as a successful means to interface circuit and EM formulations. In the classical PEEC approach, conductor cross sections are divided into volumetric filaments of rectangular cross section. Equivalent inductance, capacitance, and resistance matrices are computed for EM interactions between these filaments. These matrices are then imported into SPICE or similar circuit solvers, either directly or in the form of reduced-order models. The coupled system can then be solved in the time or frequency domain.

Owing to recent developments in systems-on-chip and in high-speed integrated circuits, there is a need to go beyond the classical filament-based PEEC for a variety of reasons. First, there is a need to model arbitrarily shaped structures including on-chip inductors, micro-electro-mechanical devices, on-chip antenna structures, et cetera. Second, at high frequencies, skin effect modeling is crucial. The classical filament-based approach addresses this by mimicking the behavior of the conduction current at these frequencies through appropriate lateral meshing of filaments. Unfortunately, such an approach is both heavily frequency dependent and potentially computationally intensive. An elegant approach to modeling skin impedance is presented by the use of surface impedance approximations in the surface-based method of moments (MoM).

The work presented in this Letter aims at using surface-only formulations utilizing surface meshes [5,7,8] in order to model more complex current distributions and their field effects arising from nonrectangular conducting structures. The surface-based version of PEEC is first presented, in a SPICE-compatible form. The circuit-solver-free equivalent coupled matrix formulation is then presented. Finally, it is shown that, by a reformulation of unknowns, the surface-based formulation can be reformulated as a modified method of moments (MoM) system.

2. SURFACE-BASED COUPLED EM-CIRCUIT SIMULATION

Conducting structures are analyzed with the use of the electric field integral equation formulation (EFIE), wherein the surface current $\mathbf{J}$ satisfies the equation

$$j\omega \frac{\mu}{4\pi} \int_s \frac{J(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' + (\nabla\phi)(\mathbf{r}) = -ZJ(\mathbf{r}) \tag{1}$$

and the scalar potential ϕ and surface charge density ρ are related through the equation

$$\phi = \frac{1}{4\pi\varepsilon} \int_s \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds'. \tag{2}$$

In Eq. (1) Z represents the surface impedance

$$Z = \sqrt{\frac{j\omega\mu}{2\sigma}} \tag{3}$$

in terms of the circular frequency ω, permeability μ, and conductivity σ. The surface impedance approximation is valid at frequencies where the skin depth is smaller than the cross section of conductors. At lower frequencies, a coupled interior EFIE utilizing lossy medium Green's functions is required. For surface current and charge modeling, the standard RWG functions $\boldsymbol{f}$ [3,4], defined over pairs of triangles are used in conjunction with a triangular tessellation of the conductor surfaces. Upon testing the EFIE, the following matrix equations are obtained:

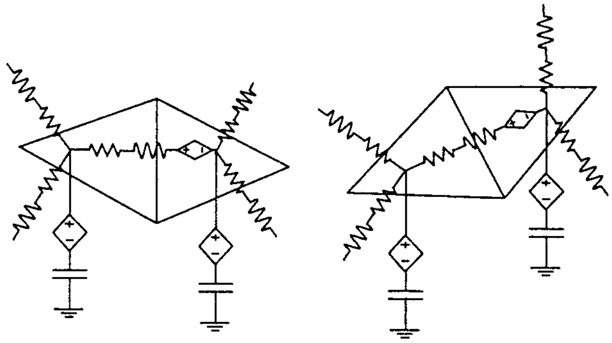

Figure 1 Equivalent circuit construction on a triangular mesh, showing inductances, resistances, capacitances, and dependent controlled sources

$$(j\omega\bar{\mathbf{L}} + \bar{\mathbf{Z}})\,\mathbf{J} = \mathbf{I} \tag{4}$$

$$\bar{\mathbf{P}}\mathbf{Q} = \mathbf{V}. \tag{5}$$

In these equations, the variables are the current, charge, and potential, and the excitation includes known terminal voltages v or current sources. The inductance, potential, and surface impedance matrices can be defined as

$$L_{ij} = \frac{\mu}{4\pi}\int_s\int_s \frac{\mathbf{f}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,ds' \cdot \mathbf{f}_i(\mathbf{r}')\,ds, \qquad i, j = 1, \ldots, N_e, \tag{6}$$

$$P_{ij} = \frac{1}{4\pi\varepsilon}\int_s\int_s \frac{\nabla' \cdot \mathbf{f}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\nabla \cdot \mathbf{f}_i(r)\,ds\,ds', \qquad i, j = 1, \ldots, N_p, \tag{7}$$

$$Z_{ij} = Z\int \mathbf{f}_i(\mathbf{r}) \cdot \mathbf{f}_j(\mathbf{r})\,ds, \qquad i, j = 1, \ldots, N_e, \tag{8}$$

where the indices traverse the total number of edges or patches, and the surface impedance matrix is nonzero for only those entries where RWG bases i and j share a common triangle. The matrix $\bar{\mathbf{A}}$ is a sparse, rectangular adjacency matrix that couples edges to triangular patches. Each row has two nonzero entries.

Such an approach can be used to incorporate EM effects, within the limitations of SPICE solvers and reduced-order models, through element stamps and controlled sources for mutual coupling terms. The distributed equivalent circuit based on triangular surface meshes is depicted in Figure 1, and Figure 2 shows the PEEC-like equivalent circuit for two neighboring triangular elements.

As discussed in the previous section, an alternative desirable approach is to remove the dependency on SPICE-like solvers, and to formulate the equations as a coupled system amenable to fast iterative or direct solution.

3. Coupled Method-of Moments-Formulation

The EFIE described in the previous section can be explicitly written in a coupled form outside SPICE in the following manner

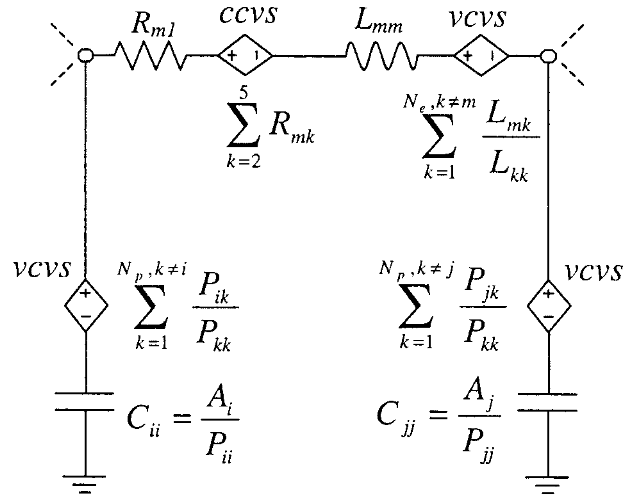

Figure 2 Equivalent PEEC-like circuit for two adjacent triangular mesh elements

$$\begin{pmatrix} j\omega\bar{\mathbf{L}} + \bar{\mathbf{Z}} & \bar{\mathbf{A}} & \bar{\mathbf{0}} & \bar{\mathbf{X}} \\ \bar{\mathbf{0}} & \bar{\mathbf{I}} & \bar{\mathbf{P}} & \bar{\mathbf{0}} \\ -\bar{\mathbf{A}}^T & \bar{\mathbf{0}} & j\omega\bar{\mathbf{D}} & \bar{\mathbf{0}} \\ \bar{\mathbf{X}}^T & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \overline{\mathbf{MNA}} \end{pmatrix} \begin{pmatrix} \mathbf{J} \\ \mathbf{V} \\ \mathbf{Q} \\ \mathbf{ckt} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{ckt\_ex} \end{pmatrix}. \tag{9}$$

In this formulation, the sparse matrix $\bar{\mathbf{X}}$ is used to enforce KVL, KCL, and field continuity. The sparse block $\overline{\mathbf{MNA}}$ represents the modified nodal analysis conductance matrix corresponding to circuit unknowns (**ckt**). The excitation includes voltage and current sources within the excitation vector **ckt_ex**. The matrix $\bar{\mathbf{D}}$ is a diagonal matrix used to enforce the current and charge continuity equation. The unknowns in this formulation include the surface current, potential, charge, and MNA circuit unknowns. Electric field excitation can also be included in this formulation, in the first block of the right-hand side. The above formulation is analogous to the filament-based PEEC method, but for a surface-only formulation. It is interesting to note that these equations can be reorganized and eliminated to obtain a simpler set of equations:

$$\begin{pmatrix} j\omega\bar{\mathbf{L}} + (j\omega)^{-1}\bar{\mathbf{A}}\bar{\mathbf{P}}\bar{\mathbf{D}}^{-1}\bar{\mathbf{A}}^T & \bar{\mathbf{X}} \\ \bar{\mathbf{X}}^T & \overline{\mathbf{MNA}} \end{pmatrix} \begin{pmatrix} \mathbf{J} \\ \mathbf{ckt} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{ckt\_eq} \end{pmatrix}. \tag{10}$$

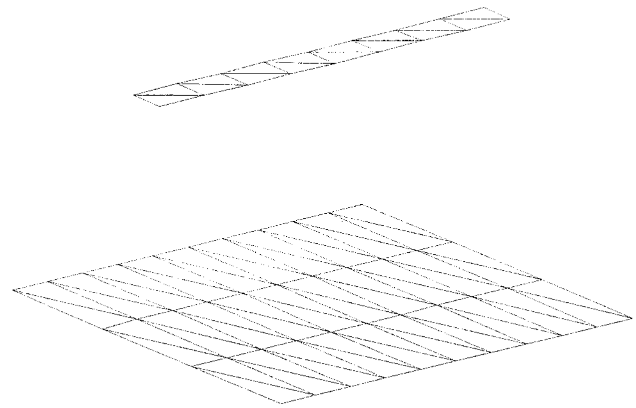

Figure 3 Interconnect over ground plane. The ground plane is 2 cm long and 1 cm wide, the interconnect is 2 cm long and 1 mm wide, and is 0.5 mm above the ground plane

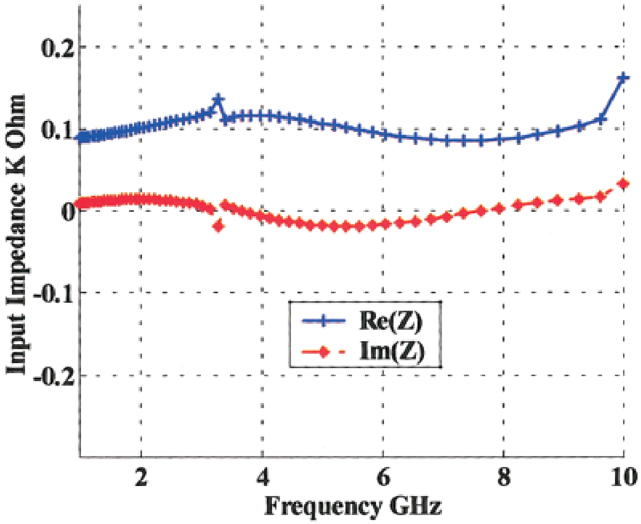

Figure 4 Input impedance of the interconnect over a ground plane as a function of frequency [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

The first block of equations above is nothing but a statement of the MoM, with appropriate interfaces (voltages and currents) to ckt unknowns. The second block is the self-consistent set of circuit equations and independent excitations. Thus the surface-based coupled formulation can be reinterpreted as a classical MoM with appropriate excitation and circuit boundary conditions.

4. SIMULATION EXAMPLES

The first example (Figure 3) is that of an interconnect over a ground plane, as in [6]. The frequency dependence of the input impedance is shown in Figure 4. It is interesting to note that the transmission-line resonance behavior of this structure is also captured. The finite-sized impedance peaks are due to coarse frequency sampling at resonance.

The next example demonstrates the ability to model time-domain cross talk. Figure 5 shows two sets of traces over a ground plane, one set running parallel, and the other set with one trace having two right-angled discontinuities and with a larger average distance between the traces. One of the traces is excited with a 600-ps duration trapezoidal pulse. The far-end cross-talk waveforms on the unexcited traces are shown in Figure 6. As expected, the closer-in trace shows a large noise waveform. Also, the separation between the two large pulses, signifying the initiation and

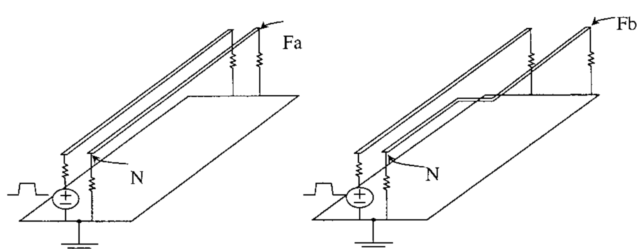

Figure 5 Pair of traces over a ground plane: straight, parallel traces (left), and bent traces (right). The ground plane is 2 cm long and 1 cm wide, the traces are 2 cm long 1 mm wide, and are 0.5 mm above the ground plane. In the left figure the traces are 1 mm apart; in the right figure the traces are 1 mm apart at the near end and 2 mm apart at the far end

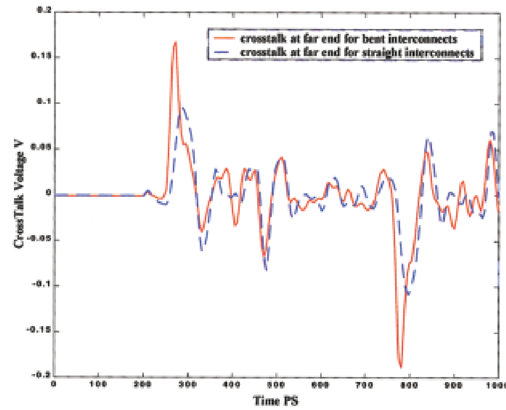

Figure 6 Far-end cross-talk voltage waveforms for the straight and bent pair of interconnects [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

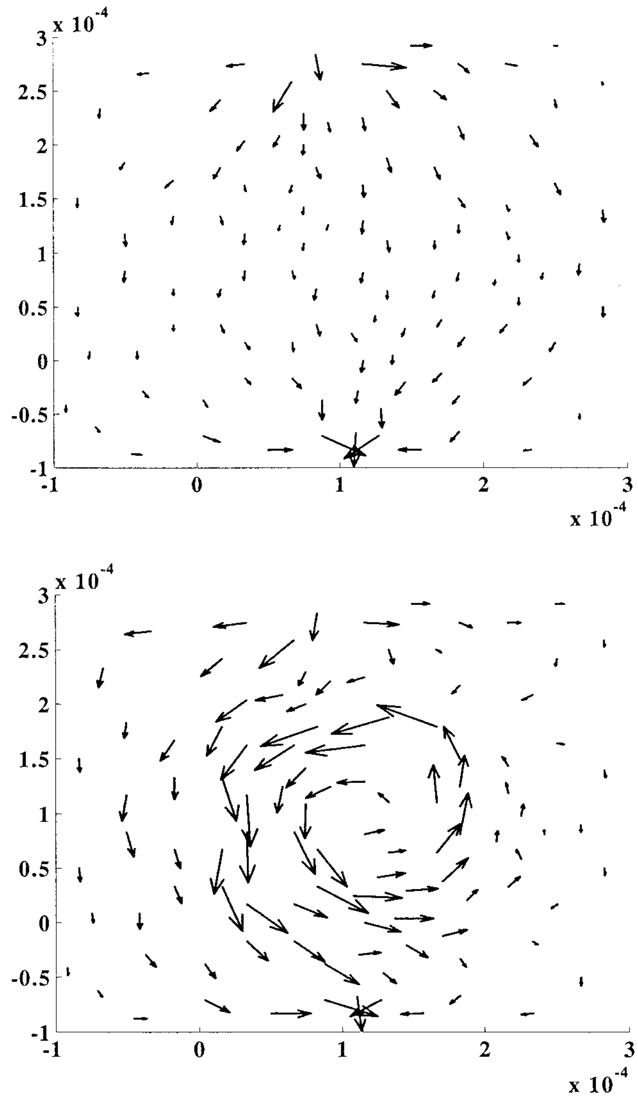

Figure 7 Induced ground plane current under spiral inductor, obtained with the use of equivalent surface-PEEC elements in SPICE, at 1 kHz (top) and 1 GHz (bottom). Spiral inductor is 200 μm $\times$ 200 μm and 30 μm above the ground plane, and the ground plane is 400 μm $\times$ 400 μm

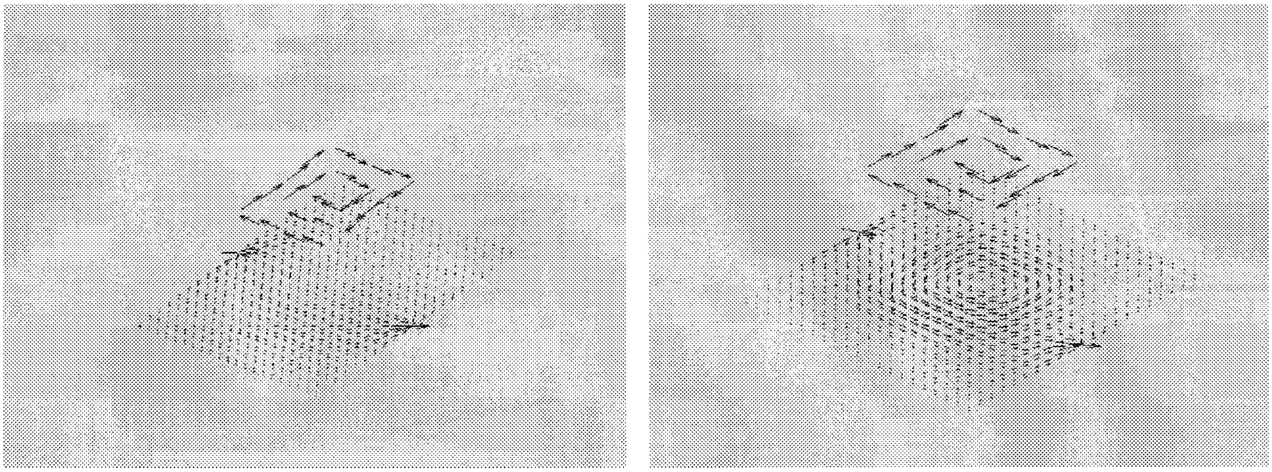

Figure 8 Induced current on ground plane under inductor, obtained with the use of the coupled-matrix solution, at 100 kHz (left) and 1 GHz (right). Also shown is the inductor current

conclusion of the trapezoidal pulse influence (through its derivative), is 600 ps.

The third example illustrates the ability of the coupled method to predict induced current behavior. A planar, spiral inductor over a ground plane is excited with a voltage source connected to the ground, and terminated with a resistance to an oppositely located point on the ground plane. At a frequency of 1 KHz, the current on the ground plane (Figure 7) is well spread out and travels along low-impedance paths from the terminal back to the source. However, at a higher frequency of 1 GHz, the inductance minimization phenomenon is observed. The current tries to flow in loops of small area in order to minimize inductive impedance at this high frequency. Although this is captured reasonably well by the SPICE-linked version of the surface-PEEC algorithm presented here, the example also highlights the strengths of the coupled matrix method. The presence of a dense matrix in SPICE makes the SPICE solution time large. However, with a dedicated dense iterative solver and coupled-matrix system outside SPICE, the overall solution is rendered far more efficient. Consequently, more resolved surface meshes can be used, and a smoother current distribution is obtained, as in Figure 8. Both high- and low-frequency behavior of the current below the inductor is exhibited in this figure.

5. CONCLUSIONS

A new surface-based coupled EM-circuit formulation is described for arbitrarily shaped three-dimensional conducting structures. The formulation is expressed first as a modified surface-based PEEC and is solved by introducing equivalent circuits in SPICE. Next it is shown that a coupled matrix system can be generated outside SPICE, which can be reduced to a modified MoM system, and can be solved efficiently with the use of an iterative solver. The method is general enough to be applicable to a wide variety of high-frequency and high-speed integrated circuit and packaging applications.

Continuing work is aimed at incorporating a fast multilevel iterative solver, multilayered and finite dielectrics, and specialized quadrature rules for very low frequency characterization of the interior lossy medium problem. A fast direct solution based on a multilevel low-rank decomposition algorithm is also being implemented.

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