A Time Domain Surface Integral Technique for Mixed Electromagnetic and Circuit Simulation

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Abstract
A full-wave time domain surface integral approach to coupled electromagnetic and circuit simulation is presented in this paper. In particular, non-linear circuit elements and effect of interference and crosstalk can be modeled in the time domain. The coupling of lumped elements to a surface integral formulation is detailed. Losses are modeled with an efficient recursive convolution.

1 Introduction

Time domain electromagnetic-circuit simulation is of interest to digital and analog-RF designers, and is essential in the case of non-linear elements and broadband coupling. In this work we propose a time-domain surface-only form that models losses using a recursive formulation of surface impedance. Using an implicit method ensures stability of the scheme. Furthermore, lumped linear and non-linear elements can be directly incorporated into this formulation through a rigorous coupling scheme for both open and closed conducting structures. The approach may be interpreted as a time-domain version of the partial element electric circuit (PEEC) [1,2] method, with the significant distinctions that the solution uses well-established method of moment (MoM) forms, is surface-only (no volumetric discretization required), makes no a priori assumptions on current flow directions. Examples include validation against time-domain PEEC, as well as a demonstration of coupling and non-linear effects.

2 Integral Equation Formulation

For a lossy conductor, the following time domain integral equation (TDIE) holds under a surface impedance approximation

$$\left[\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t}+\nabla\Psi(\mathbf{r},t)\right]_{\tan}=\left[-Z_s(t)*\left(\frac{\partial \mathbf{J}(\mathbf{r},t)}{\partial t}\right)\right]_{\tan}+\mathbf{E}^i(\mathbf{r},t)\Big|_{\tan} \tag{1}$$

where $\mathbf{A}(\mathbf{r},t)$ is the vector potential, $\Psi(\mathbf{r},t)$ is the scalar potential, Z_s is the time-domain surface impedance, **J** is the equivalent surface current density, and $\mathbf{E}^i(\mathbf{r},t)$ is incident electric field or its equivalent due to a circuit excitation. For a homogeneous medium the potentials are given by

$$\mathbf{A}(\mathbf{r},t)=\mu\int_S\frac{\mathbf{J}(\mathbf{r}',t-R/c)}{4\pi R}ds',\Psi(\mathbf{r},t)=\frac{1}{\varepsilon}\int_S\frac{q_s(\mathbf{r}',t-R/c)}{4\pi R}ds' \tag{2}$$

where the domain S is the surface of the conductor, and the surface impedance is given by $Z_s(t)=\sqrt{\mu/\pi\sigma t}$ where μ, ε, and σ are the material parameters. The surface charge density q_s is related to **J** through the continuity equation $q_s(t)=-\int_{\tau=0}^{t}\nabla\cdot\mathbf{J}d\tau$. In this work the TDIE (1) is formulated and solved in the method of moments (MoM) form using the surface-triangle-pair based Rao-Wilton-Glisson (RWG) basis functions [3,4].

3 Efficient Loss Modeling in the Time Domain

The surface impedance model leads to a convolution for each time step, which can become prohibitively expensive. To circumvent this, we adopt the following fast recursive formulation, presented in [5]. The convolution involving the surface impedance is

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$$Z_s(t)*(\frac{\partial J(\mathbf{r},t)}{\partial t}) = \int_0^t \sqrt{\frac{\mu}{\pi\sigma\tau}} \frac{\partial \mathbf{J}(\mathbf{r},t-\tau)}{\partial(t-\tau)} d\tau = \sqrt{\frac{\mu}{\pi\sigma}} \int_0^t \frac{1}{\sqrt{\tau}} \frac{\partial \mathbf{J}(\mathbf{r},t-\tau)}{\partial(t-\tau)} d\tau \tag{3}$$

After applying finite difference in time and some algebraic manipulation, (3) can be rewritten as

$$\sqrt{\frac{\mu}{\pi\Delta t\sigma}} \sum_{m=0}^{n-1} Z_0(m)[\mathbf{J}(\mathbf{r},(n-m)) - \mathbf{J}(\mathbf{r},(n-m-1))] \quad ; \quad Z_0(m) = \int_m^{(m+1)} \frac{d\alpha}{\sqrt{\alpha}} \tag{4}$$

where the indices m and n specify present and cumulative time steps. Finally, the function in (4) can be accurately represented as a sum of complex exponentials as $Z_0(m) = \int_m^{m+1} \frac{1}{\sqrt{\alpha}} d\alpha \cong \sum_{i=1}^{N} a_i e^{m\beta_i}$ where the coefficients and exponential weights are determined by Prony's method. It is shown in [5] that N=10 suffices to produce very high accuracy approximations for the given function. This enables (4) to be rewritten as

$$\sqrt{\frac{\mu}{\pi\sigma\Delta t}} \sum_{i=1}^{N} \sum_{m=0}^{n-1} a_i e^{m\beta_i} [\mathbf{J}(\mathbf{r},(n-m)) - \mathbf{J}(\mathbf{r},(n-m-1)] = \sqrt{\frac{\mu}{\pi\sigma\Delta t}} \sum_i^N \mathbf{\Phi}_i^n \tag{5}$$

Finally, a recursive relationship can be obtained between $\mathbf{\Phi}_i^n$ and $\mathbf{\Phi}_i^{n-1}$

$$\mathbf{\Phi}_i^n = a_i[\mathbf{J}(\mathbf{r},n) - \mathbf{J}(\mathbf{r},(n-1))] + e^{\beta_i}\mathbf{\Phi}_i^{n-1} \tag{6}$$

This permits a single computation at each time step instead of a convolution, thus enabling rapid loss computation along with the regular TDIE time stepping.

4 Coupling Circuit and Electromagnetic Equations

A critical part of the use of TDIE for circuit problems is the proper form of excitation, based on circuit sources. Moreover, compatibility and interference modeling also requires field excitation that standard in electromagnetic integral equation forms. For the circuit excitation, we follow the approach proposed in [6] and [7] in the frequency domain, and extend it to the time domain here. To summarize the approach, two distinct coupling schemes are proposed (Fig. 1). The first is for open or zero thickness structures wherein circuit current feeds into edges of the triangular mesh. In this instance, current continuity, along with the use of half-RWG bases is sufficient; in physical terms half-RWG bases are equivalent to inductors projecting out from each terminal edge, which are then connected equi-potentially to a circuit node. In the second case, a circuit current is introduced to a closed equivalent surface. One or more triangles may be considered to be part of the terminal. In this case, the continuity equation needs to be modified to include the current due to the circuit I_c : $\nabla \cdot \vec{J} + \frac{\partial q_s}{\partial t} = \frac{I_c}{A_p}$ where the denominator denotes the area of the terminal, which may contain one or more triangles. This new source creates a new scalar potential through modification of the total charge and divergence of current. The potentials of terminal triangles are tied to the circuit node potential.

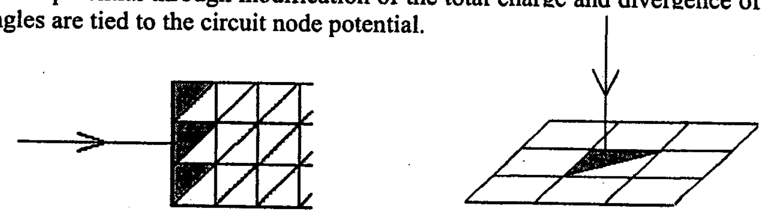

Figure 1. Circuit-EM mesh interconnections for open (left) and closed (right) structures

Based on the above coupling mechanism a coupled matrix system is established as following.

$$\begin{bmatrix} \overline{\mathbf{EM}} & \overline{\mathbf{EC}} \\ \overline{\mathbf{EC}^T} & \overline{\mathbf{MNA}} \end{bmatrix} \begin{bmatrix} \mathbf{J} \\ \mathbf{CIR} \end{bmatrix} = \begin{bmatrix} \mathbf{HISTORY} + \mathbf{E\_FIELD} \\ \mathbf{SOURCE} \end{bmatrix} \tag{7}$$

The **EM** block in the coupled matrix represents the MoM section. The sparse **EC** block is the connectivity between the EM and the circuit part. The **MNA** block represents the modified nodal analysis conductance matrix corresponding to the circuit unknowns (**CIR**). The **J** vector represents the surface unknowns on the conductors. he **HISTORY** represents influence from past times, and the **E_FIELD** vector is the electrical field incident to the circuit. The **SOURCE** vector contains voltage or current sources applied to the circuit.

5 Numerical Results

In the first example, common mode current is computed for the structure In Fig. 2. A wire is placed between two ground strips, where L1 and L2 are 2cm and 0.4cm long respectively. The gap between the wire and the ground strip is 0.5mm. The wire and the two ground strips have the same width of 1mm. Trapezoidal voltage pulses with 1V amplitude, and rising, roof and falling time of 100ps are applied as shown. A 10Ω resistor is placed between the source and the ground. Because of the different length of the two ground strips, there is a common mode component in the ground currents, which is evident in Fig. 3. This result matches well with the published result for the same structures in [8].

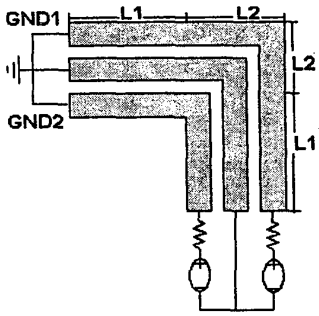

Figure 2. Wire between two ground strips

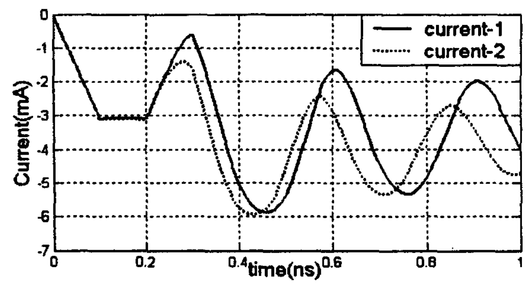

Figure 3. Current through GND1 and GND2

The second example deals with interference on a two-level multiple crossover circuit (Fig. 4) comprised of lossy interconnects with conductivity 5.8e+6 S/m, with non-linear circuit elements. Interconnects on the upper level are $40\mu m$ wide, 0.75mm long, with a $40\mu m$ separation between them. Interconnects on the lower level are $50\mu m$ wide, 0.68mm, with a $50\mu m$ separation between them. The distance between the two levels of interconnect is $50\mu m$. Each port at the upper level, except ports 2 and 4, is terminated by a 10pF capacitor. Ports 2 and 4 are terminated by diodes with characteristics Is=5.e-9A and VT=25.e-3V. Each port at the lower level is terminated by a $200\,\Omega$ resistor. A 1V sinusoidal voltage source $V(t) = \sin(\omega t)$, where $\omega = 2\pi f$, $f = 10\,GHz$ is applied to port 1. A plane wave with $\mathbf{k} = +\hat{\mathbf{z}}$, $\mathbf{E}(t) = -1000\sin(\omega_0 t)(V/m)\hat{\mathbf{y}}$, where $\omega_0 = 2\pi f_0, f_0 = 5GHz$, is incident to the nonlinear circuit. The output with no interfering plane wave, at port 2, as well as crosstalk at port 4, is shown in Fig. 5. Once the interference is introduced, the output distorts as shown in Fig. 6, and the crosstalk also changes. What is interesting is the non-linear effect of harmonic coupling, where the harmonic of the plane wave at 10GHz couples into the circuit, causing distortions at the circuit frequency of 5GHz. This is observed in Figs. 7 and 8, where the FFT's of the uncoupled and distorted signals are depicted.

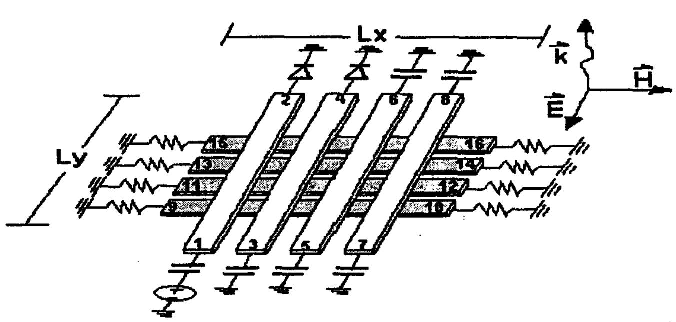

Figure 4. Incident field coupling to nonlinear circuit

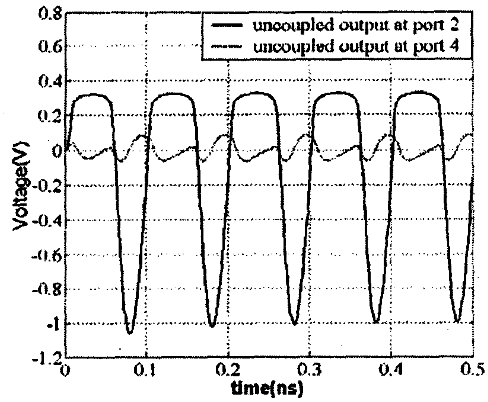

Figure 5. Uncoupled output at port 2

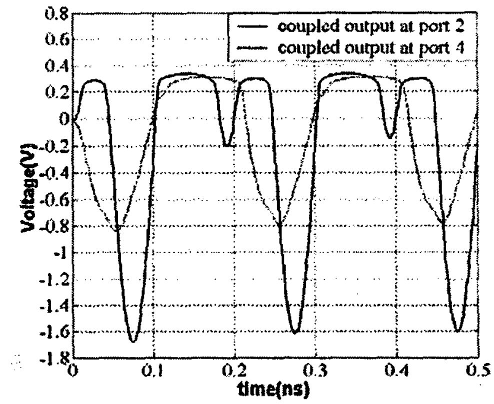

Figure 6. Coupled output at port 4

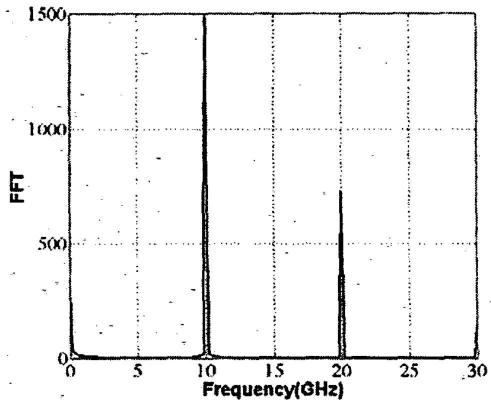

Figure 7. FFT magnitude of uncoupled output at port 2

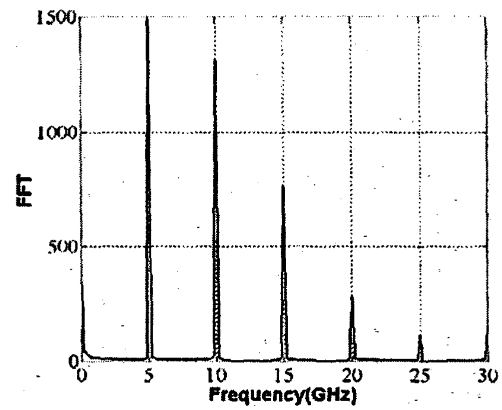

Figure 8. FFT magnitude of coupled output at port 2

6 Conclusions

A time domain, surface based, full-wave coupled-EM and circuit matrix system was developed. Loss was modeled with a recursive convolution. Coupling schemes were discussed and examples were shown that included common mode currents, non-linear circuits, and coupling to interference. Current and future work includes adaptive time stepping, mesh refinement, and fast solvers.

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References

[1] A. E. Ruehli, "Equivalent circuit models for three dimensional multiconductor systems," *IEEE Trans. Microwave Theory Tech.*, vol. 22, pp. 216–221, Mar. 1974.

[2] P.J. Restle, A.E. Ruehli, S.G. Walker, and G. Papadopoulos, "Full-wave PEEC time-domain method for the modeling of on-chip interconnects," *IEEE Trans. Comp.-Aided Des. of Integ. Circuits Systems*, vol. 20, no. 7, pp. 877-886, July 2001.

[3] S. M. Rao and D. R.Wilton, "Transient scattering by conducting surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 56–61, Jan. 1991.

[4] S. M. Rao, *Time Domain Electromagnetics*, Academic Press, San Diego, CA, 1999.

[5] K.S. Kunz and R.J. Luebbers, *The Finite Difference Time Domain Method in Electromagnetics*, CRC Press, Boca Raton, FL, 1993.

[6] Y. Wang, V. Jandhyala, and R. Shi, "Coupled electromagnetic-circuit simulation of arbitrarily-shaped conducting structures," *Proceedings of Electrical Performance of Electronic Packaging*, pp. 233-236, 2001.

[7] V. Jandhyala, Y. Wang, D. Gope, and R. Shi, "A surface-based integral equation formulation for coupled circuit-electromagnetic simulation," *Microwave Optical Technology Letters*, July 20, 2002.

[8] W. Pinello, A. Ruehli, A. Cangellaris, "Analysis of interconnect and package structures using PEEC models with radiated emissions," *Proc. IEEE Int. Symp. Electromagnetic Compatibility*, pp. 353-358, 1997.