A Padé via AWE Fast Frequency Sweep for Quasi-static Coupled Electromagnetic and Circuit Simulation

Todd West and Vikram Jandhyala
Department of Electrical Engineering, University of Washington, Seattle, WA 98195
Phone: (206) 543-2186, Fax: (206) 543-3842, Email: {twest,jandhyala}@ee.washington.edu

1 Introduction

Padé via AWE (Asymptotic Waveform Expansion) [1] and other model order reduction techniques such as PVL (Padé via Lanczos) [2] and PRIMA [3] have been used for efficient solution of a wide variety of problems. PVL and PRIMA use Lanczos and Arnoldi processes, respectively, to extract the dominant eigenvalues of system matrices of the form $\overline{\mathbf{G}} + s\overline{\mathbf{C}}$. Extending PVL or PRIMA to handle system matrices with other functional forms is difficult for a number of reasons [4]. AWE based methods do not suffer from some of these limitations, so EM (electromagnetic) MOR (model order reduction) has focused on applying AWE to various types of problems. Of particular interest is treatment of the system matrix as the superposition of several other matrices that form a polynomial in frequency [5].

This work considers a coupled EM-circuit system [6] where the EM portion of the system is expressed using boundary element integral equations represented by the method of moments (MoM), and the circuit portion is described using modified nodal analysis (MNA). The system therefore contains two different kinds of physics. The advantage of the coupled approach is the ability to observe circuit effects on EM quantities, and to design EM-aware microelectronic circuits. Here we show that Padé via AWE can be used successfully to implement fast frequency sweeps for coupled EM-circuit systems.

2 Single Point AWE Expansion and Padé Approximation

As described in [6], coupled EM-circuit simulation can be performed by forming the system

$$\begin{bmatrix} \overline{\mathbf{Z}}_{11} & \overline{\mathbf{Z}}_{12} & \overline{\mathbf{0}} \\ \overline{\mathbf{Z}}_{21} & \overline{\mathbf{Z}}_{22} & \overline{\mathbf{C}} \\ \overline{\mathbf{0}} & \overline{\mathbf{C}}^T & \overline{\mathbf{MNA}} \end{bmatrix} \mathbf{x} = \mathbf{z},\ \mathbf{x} = \begin{bmatrix} \mathbf{J} \\ \mathbf{I}_c \\ \mathbf{ckt} \end{bmatrix},\ \mathbf{z} = \begin{bmatrix} \mathbf{ex}_{em} \\ \mathbf{0} \\ \mathbf{ex}_{ckt} \end{bmatrix} \tag{1}$$

where $\overline{\mathbf{Z}}_{11}$ is the pure electromagnetic part of the system and $\overline{\mathbf{Z}}_{12}$, $\overline{\mathbf{Z}}_{21}$, and $\overline{\mathbf{Z}}_{22}$ extend the electromagnetic system to provide terminals where the circuit part of the system, $\overline{\mathbf{MNA}}$, can be coupled to the electromagnetic part. $\overline{\mathbf{C}}_{12}$ expresses the coupling between the electromagnetic and circuit systems. $\mathbf{J}$ is a vector of electromagnetic unknowns and $\mathbf{ckt}$ is the voltage and current unknowns in the circuit part of the system. $\mathbf{I}_c$ is the unknowns for the EM connection points. $\mathbf{ex}_{em}$ is the electromagnetic excitation of the system and $\mathbf{ex}_{ckt}$ is a circuit excitation vector containing independent voltage and current sources. Such a system enables fully-coupled simulation as well as terminal-based EM models for use in circuit simulation.

For the EM portion of the system, we use the electric field integral equation (EFIE) with surface impedance. The EFIE enforces tangential continuity of electromagnetic fields through

$$-\langle j\omega \mathbf{A}(\mathbf{J}_s, \mathbf{r})\rangle - \left\langle \nabla\phi\left(\frac{-\nabla\cdot\mathbf{J}_s}{j\omega}, \mathbf{r}\right)\right\rangle - \left\langle Z_s\left(\sqrt{\omega}\right)\mathbf{J}_s(\mathbf{r})\right\rangle = -\langle \mathbf{E}_{incident}(\mathbf{r})\rangle \tag{2}$$

0-7803-8302-8/04/$20.00 ©2004 IEEE

where **r** is some position on the surface of a conductor, **A** is the magnetic vector potential in the Lorentz gauge, ϕ the electric scalar potential, $\mathbf{Z}_s$ the surface impedance, **J**, the surface current, and $\mathbf{E}_{incident}$ the incident electric field strength. For electrically small systems, such as mixed-signal systems-on-chip, the quasi-static Green's function applies. **A** is therefore frequency independent, $\nabla\phi$ is inversely proportional to frequency, and $\mathbf{Z}_s$ is proportional to the square root of frequency. Similarly, the circuit part of the system is

$$\overline{\mathbf{MNA}} = \frac{\overline{\mathbf{Y}}_L}{f} + \overline{\mathbf{Y}}_R + f\overline{\mathbf{Y}}_C \tag{3}$$

where $\overline{\mathbf{Y}}_L$, $\overline{\mathbf{Y}}_R$, and $\overline{\mathbf{Y}}_C$ are the admittances associated with inductive, resistive, and capacitive elements in the circuit portion of the system, respectively. In turn, the coupled system matrix is expressed as the sum of frequency dependent matrices

$$\begin{bmatrix} \overline{\mathbf{Z}}_{11} & \overline{\mathbf{Z}}_{12} & \overline{\mathbf{0}} \\ \overline{\mathbf{Z}}_{21} & \overline{\mathbf{Z}}_{22} & \overline{\mathbf{C}} \\ \overline{\mathbf{0}} & \overline{\mathbf{C}}^T & \overline{\mathbf{MNA}} \end{bmatrix} = \frac{\overline{\mathbf{C}}_f}{f} + \overline{\mathbf{R}} + \sqrt{f}\,\overline{\mathbf{Z}}_s + f\overline{\mathbf{L}} \tag{4}$$

Rewriting in terms of $g = \sqrt{f}$ yields the system equation

$$\left(\overline{\mathbf{C}}_f + g^2\overline{\mathbf{R}} + g^3\overline{\mathbf{Z}}_{Z_s} + g^4\overline{\mathbf{L}}\right)\mathbf{x} = g^2\mathbf{z} \tag{5}$$

We then expand $\mathbf{x}$ in a Taylor series of k terms about $g_0 = \sqrt{f_0}$ with $g = g_0 + \gamma$, using a conditioning factor ξ as in [2], and derive the AWE recursion relationships as in [6] to find the AWE moments $\mathbf{x}_0 \ldots \mathbf{x}_k$. A vector Padé approximation $\hat{\mathbf{x}}_{Padé}$ is then computed for $\mathbf{x}$ by forming and solving the Padé Hankel matrix for denominator coefficients of each element in $\mathbf{x}$ and using the direct formula for numerator coefficients ([7], Chapter 1, equations 1.6 and 1.7).

3 Multipoint Expansion and Numerical Conditioning

Typically, the frequency range of interest is known *a priori*, and it is desired to find an approximation or set of approximations which is accurate over the entire range. We use a binary search which places the Padé expansion point in the middle of the frequency range. The resulting expansion is evaluated at the interval's endpoints and compared to the exact solution at the endpoints. If the approximation's error is larger than some specified tolerance, the interval is divided into two sub intervals of equal size, new expansions performed, and checked against exact solutions at their intervals' endpoints. If necessary, additional subdivision is performed recursively until the desired accuracy is obtained.

Calculation of Padé approximants directly from AWE moments is notoriously poorly conditioned [7]. The scaling factor ξ used in (7) has great impact on the Padé Hankel matrix's conditioning. A variety of possible choices for ξ have been proposed [1]; our current method is to compute an initial $\mathbf{x}_1$ with ξ=1 and then update ξ as

$$\xi = \mathrm{mean}(\mathbf{x}_1 ./ \mathbf{x}_0) \tag{6}$$

where ./ denotes element by element division and near zero elements in $\mathbf{x}_0$ are ignored. $\mathbf{x}_1 \ldots \mathbf{x}_k$ are then computed with the new value of ξ. Such single step adaptation of ξ is not, in principle, optimum. However, effective values of ξ have reliably been produced using this method. The authors have also implemented the WCAWE method of [5] and found WCAWE and Padé via AWE with adaptive ξ produce near identical results on a variety of test cases.

Each node present in $\overline{\mathbf{MNA}}$ whose voltage is invariant in frequency results in an exact AWE expansion with $x_{i,0} = V_i$ and $x_{i,1} \ldots x_{i,k} = 0$, where V_i is the node's voltage. The Hankel matrix is thus singular for all Padé approximants of order higher than zero. A similar situation arises where any unknown in the system is almost exactly a polynomial in frequency. Adaptive Padé order reduction is needed to handle such cases.

4 Numerical Results

As an example, we consider the low noise amplifier (LNA) shown in figure 1. The operating frequency of inductively decoupled LNAs, such as this design, is determined by the resonance of the output inductor with the output and device capacitances. This is a difficult case for Padé via AWE, as effectiveness of AWE decreases near resonance. For both amplifiers, the inductors are modeled using an EM MoM approach and the rest of the amplifier is represented with a circuit system. The operating points of the amplifiers are found in Spice using the inductors' DC resistances and then small signal models are evaluated for the MOSFETs and used with the Padé fast frequency sweep. Table 1 compares the time needed for Padé solution relative to that of a conventional frequency sweep which solves the system at some number of frequency points. Both Padé and the conventional sweep use only LU factorization to solve the systems. In principle, fast matrix-vector methods can further accelerate the frequency sweep, provided appropriate pre-conditioners for improving the condition number (on the order of 10^9) can be developed.

solve time for conventional frequency sweep	Padé setup time	Padé solve time for frequency sweep	reduction in solve time by using Padé with ε<0.01
100 points: 4.32 hours 250 points: 10.8 hours	6 expansions 53.5 minutes	100 points: 340ms 250 points: 850ms	100 points: 79.4% 250 points: 91.7%

Table 1 Padé simulation results

Figure 2 shows the LNA gain and the Padé accuracy as a function of the number of Padé expansions used. As a way of measuring the accuracy of a Padé expansion, we use the error norm

$$\varepsilon(f) = \frac{\left\|\hat{\mathbf{x}}_{\text{Padé}}(f) - \mathbf{x}(f)\right\|_2}{\left\|\mathbf{x}(f)\right\|_2} \tag{7}$$

5 Conclusion

A fast frequency sweep for coupled EM-circuit systems, enabling rapid characterization of circuits with EM components, has been demonstrated using a simple Padé via AWE approach. In future work, a fast direct solver [8] will be used in conjunction with the sweep to obtain rapid solution of poorly conditioned systems for realistic on-chip structures. Extension to nonlinear MOR [9] is also of interest for handling devices outside of the small signal range.

Acknowledgements

The authors thank Dicle Ozis, Sambuddha Bhattacharya, Brian Ward, and Yong Wang for their helpful input. This work was supported by DARPA-MTO NeoCAD grant N66001-01-1-8920.

References

[1] V. Raghavan, R. A. Rohrer, L. T. Pillage, J. Y. Lee, J. E. Bracken, and M. M. Alaybeyi, "AWE-Inspired," *Proceedings of the IEEE Custom Integrated Circuits Conference*, pp. 18.1.1–18.1.8, 1993.

[2] Peter Feldmann and Roland W. Freund, "Efficient Linear Circuit Analysis by Padé Approximation via the Lanczos Process," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 14, no. 5, pp. 639–649, 1995.

[3] Altan Odabasioglu, Mustafa Celik,, and Lawrence T. Pileggi, "PRIMA: Passive Reduced-Order Interconnect Macromodeling Algorithm," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 17, no. 8, pp. 645–654, 1998.

[4] Joel R. Phillips, Luca Daniel, and L. Miguel Silveira, "Guaranteed Passive Balancing Transformations for Model Order Reduction", *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 22, no. 8, pp. 1027–1040, 2003.

[5] Rodney D. Slone and Jin Fa Lee, "Well-Conditioned Asymptotic Waveform Evaluation for Finite Elements", *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 9, pp. 2442–2447, 2003.

[6] Y. Wang, D. Gope, V. Jandhyala and C.J. Richard Shi, "Integral Equation-based Coupled Electromagnetic Circuit Simulation In the Frequency Domain," *IEEE Antennas and Propagation International Symposium*, vol. 3, pp. 328-331, June 22-27, 2003

[7] George Baker and Peter Graves-Morris, *Padé Approximants*, 2nd edition. Cambridge University Press, New York, NY, USA, 1996.

[8] Dipanjan Gope and Vikram Jandhyala, "An Iteration Free Fast Multilevel Solver for Dense Method of Moment Systems", *Electrical Performance of Electronic Packaging*, pp.177–180, 2001.

[9] Michał Rewieński and Jacob White, "A Trajectory Piecewise-Linear Approach to Model Order Reduction and Fast Simulation of Nonlinear Circuits and Micromachined Devices", *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 22, no. 2, pp. 155–170, 2003.

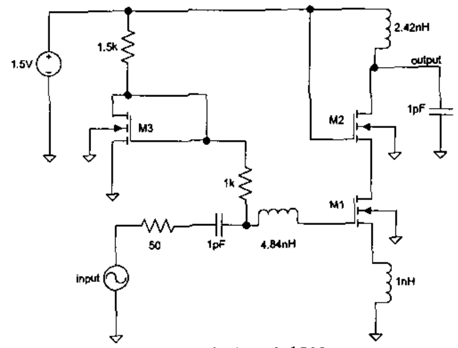

Figure 1 single ended LNA

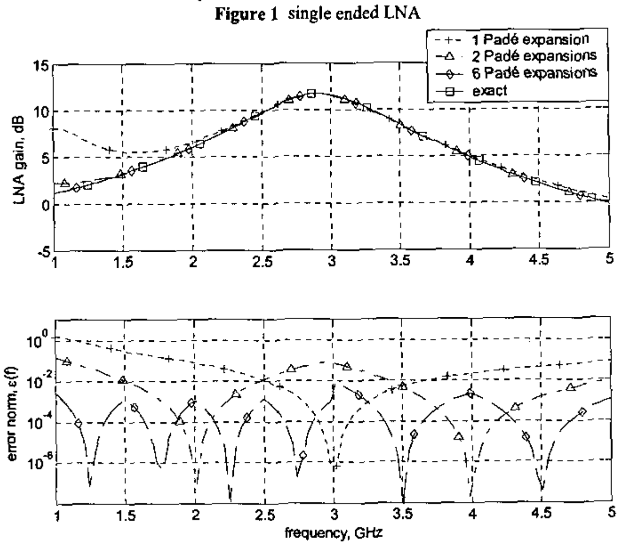

Figure 2 Padé results for single ended LNA