

An Accurate Hierarchical Electromagnetic-Circuit Technique for Statistical Analysis of RF Circuits

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1. Introduction

Modeling process variations in circuits is becoming necessary as the technology continues to shrink in size [1]. When this is coupled with increasing frequencies of operation of circuits in communication systems (>10 GHz), the use of full-wave field solvers is necessary to model the on-chip passive components such as spiral inductors and transformers. Thus it becomes necessary to couple full-wave field solvers and SPICE-like simulators for accurate statistical analysis of RF circuits. The present paper reviews the issues associated with using circuit models for electromagnetically modeled passive elements in conjunction with SPICE for statistical analysis. It then proposes a method which circumvents the outlined limitations.

2. Challenges in statistical analysis using circuit models for EM objects

Field solvers are primarily used by RF designers to model the full-wave effects of on-chip passive components such as spiral inductors and transformers. The present paper focuses on spiral inductors and their use in microwave amplifiers. In a system where no variability is modeled, well known techniques exist to derive accurate circuit models for the passives modeled by field solvers and to incorporate them in SPICE simulations [2]. The scenario changes drastically when the system is subjected to variability under process variations. The parameters that are important for field solver simulations are very different from those influencing the active circuit behavior and they cannot be used directly in the present framework for statistical analysis of circuits with SPICE-like simulators.

On-chip passive elements such as spiral inductors, transformers or any structures modeled by a field solver are often represented (or “synthesized”) as circuit models. These element values are determined by means of an optimization procedure from the s-parameters obtained through field solver simulations [2]. The circuit element values are in general correlated. When such an equivalent circuit for the inductor is used with SPICE for statistical analysis of a larger circuit such as an amplifier, the need for accurate correlation matrices arises in order to guarantee the accuracy of the pdfs (probability density function) of higher-level parameters such as gain and return loss. Extracting accurate correlation information is a difficult task. Assuming independence among these random variables representing the circuit equivalent parameters leads to erroneous results. This phenomenon is first shown for a simple transmission line model where Monte-Carlo analysis is feasible and hence can be used for comparison purposes as shown in Section 3.

3. Basic versus derived variations

A transmission line example is used to illustrate the effect of how an erroneous pdf is produced by not modeling the correct correlation. The transmission line considered in this example is a section of a microstrip with a nominal Z_0 of 50Ω , terminated by a 50Ω load. The microstrip is subjected to process variations in the form of the width of the trace and the dielectric constant of the substrate. The pdf of the absolute value of the input impedance is taken as the quantity of interest. Next the propagation constant β and the characteristic impedance Z_0 which are akin to the derived circuit equivalent quantities of a spiral inductor are taken to be the independent random variables for the system. However, they are correlated since they are different functions of the same basic parameters.

In this example, the correlation coefficient between β and Z_0 is found to be -0.51 from Monte-Carlo analysis. This is exactly the quantity that is difficult to calculate for the circuit equivalents of more complicated structures such as spiral inductors. Assuming independence will produce an error in the pdf of the required objective function $\text{abs}(Z_{\text{in}})$. The variation in the pdf produced by the two types of variations is shown in Figure 1. This illustrates the fundamental problem of incorporating circuit equivalents for EM modeled objects without accounting for the correct correlation.

A black-box approach based on y-parameters eradicates the need to determine correlation matrix for the derived quantities. The transmission line section is modeled as a 2-port network characterized by its y-parameters. It is easy to compute the quantity of interest $\text{abs}(Z_{\text{in}})$ in terms of the y-parameters and the load impedance analytically. The y-parameters are then expressed as response surfaces in terms of the two underlying basic variation parameters by employing a 2nd order, full factorial design [5]. It becomes possible to hierarchically express the quantity of interest in terms of the basic process variations (which are Gaussian and independent and easy to generate samples from) and a Monte-Carlo on this closed form expression yields the pdf of the desired quantity. The equivalence of this approach with the brute force Monte-Carlo is shown in Figure 2.

3. Statistical Performance Analysis of Microwave Amplifiers

In this work, the statistical analysis of spiral inductors involves three independent variables, namely the track width of the metal layer in which the inductor is fabricated, the oxide thickness and the substrate conductivity. Physpack [2] is the full-wave solver used for the extraction of the port parameters of the inductors. PhysPack is based on fast multilevel tree-based methods for accelerated integral equation solution and therefore provides exceptional speed and memory performance, ideal for use in this application.

It is shown that the pdfs of equivalent circuit parameters such as the series inductance L and the series resistance R are non-Gaussian and correlated [4]. The same holds true for a general circuit model of the spiral inductor. Determining the correlation matrix for these elemental values is a challenge in itself and may not be readily available for all possible designs of the inductors and the non-Gaussian nature of these pdfs only worsens the problem.

The technique discussed in section 3 is extended to the statistical analysis of performance metrics of a single-ended microwave amplifier shown in Figure 3(a). The steps involved

in this methodology are described in detail in a previous work [4]. For the active part, random variations in gate oxide thickness, zero-bias threshold voltage reduction and reduction in channel length have been taken into account with appropriate correlations.

Circuit and EM objects are decoupled at their points of interconnection as illustrated in Figure 3(b). Full factorial, second order response surfaces are constructed for the magnitude and phase of each of the extracted y-parameters of the decoupled circuit and the spiral inductors. The final objective functions such as LNA gain and input reflection coefficient are expressed in terms of both the circuit and EM y-parameters. Thus the eventual objective functions are captured in terms of the basic circuit and EM parameters, which facilitate the computation of pdfs and obviate the need for optimization procedures and correlation computation. A Monte-Carlo analysis on these closed-form expressions followed by a binning procedure furnishes the pdfs of the required objective functions. Figure 4(a) and 4(b) represent the pdfs of some parameters of interest. Two different types of variations (standard deviations are 5% and 10% of the range) are highlighted. Yield measures for various qualifying criteria are shown in table I.

Table I. Yield Table

Circuit Performance	Type 1 variation	Type 2 variation
Gain > 16.5 dB	89.28%	70.45%
$S_{11} < -15$ dB	96.58%	77.85%
Overall	86.23%	50.06%

5. Conclusions

In this paper, it is shown that accurate statistical analysis of RF circuits requiring the use of a full-wave field solver can be accomplished by suitable combination of the port y-parameters. This method obviates the need for an optimization step required to find the equivalent circuit of the on-chip passive elements. Since it works with the basic electromagnetically important variables, no knowledge of the correlation information between the elements of the equivalent circuit is needed and all the correlation is inherently preserved.

6. References

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7. Figures

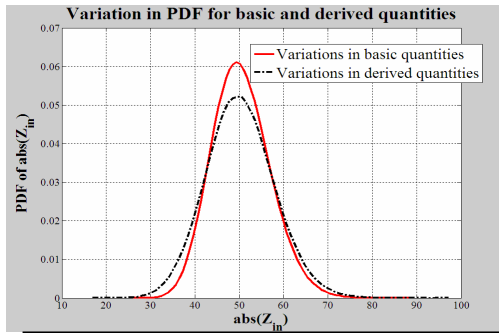


Fig. 1 Illustrating the erroneous PDF by modeling using derived variations

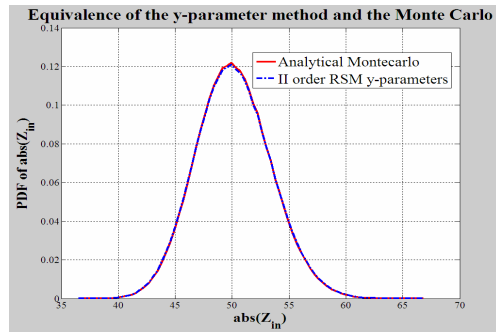


Fig. 2 Decoupled y-parameter approach vs. Monte Carlo analysis

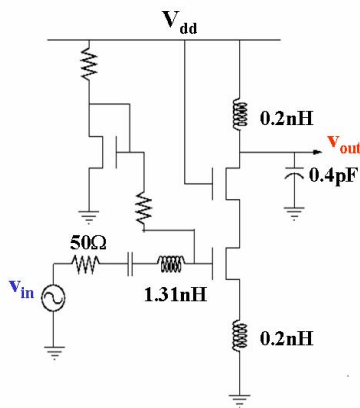


Fig. 3(a) Amplifier Schematic

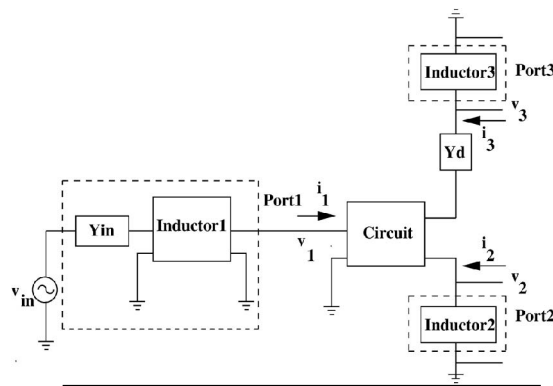


Fig. 3(b) Y-parameter port definitions, showing decoupling

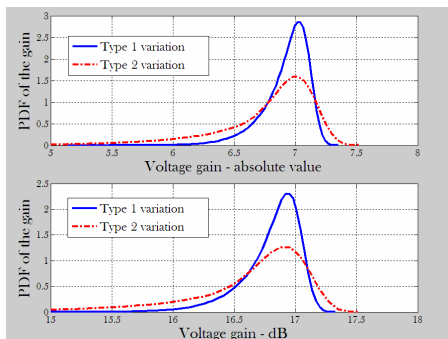


Fig. 4(a) PDF of the gain

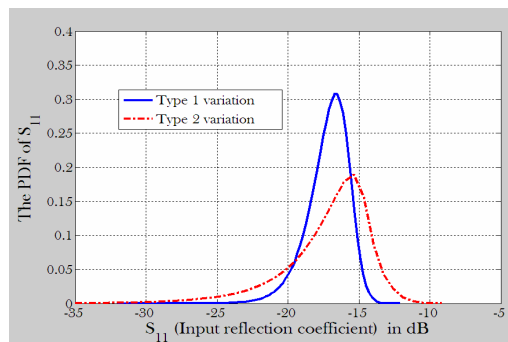


Fig. 4(b) PDF of the return loss