Statistical Study Of On-Chip Spiral Inductors Using Integral Equation Based Full-Wave Electromagnetic Analysis

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Abstract

This paper presents a statistical study of the various parameters of interest of an on-chip spiral inductor in the 10 GHz range using the PMCHWT formulation [1]. Variability in the conductivity of the CMOS grade Silicon substrate and trace metal widths are assumed. We show that EM analysis of these inductors can accurately help in understanding the impact of process variations especially at high frequencies. The use of the PMCHWT formulation also permits analysis of the electrical properties of the substrate that are particularly important at higher clock and carrier frequencies. To enable fast extraction of statistics, a simple 2D Lagrange interpolation technique is used to approximate the surfaces of the quantities of interest and then a Montecarlo operation is carried out on the interpolated 2D polynomial. We extract the Probability Density Functions (PDFs) of the effective Resistance (R), the inductance (L) and the Quality factor (Q) of a spiral on a lossy Silicon substrate.

1. Introduction

With the ever increasing upward shift in carrier frequencies, electromagnetic analysis of various on-chip components is becoming increasingly important. In particular, spiral inductors are used in many critical blocks such as LNAs, delay lines, VCOs, and transformers. In addition, modeling process variations has acquired increasing importance as we move to deep sub-micron designs [2]. Further since analog/RF technologies are moving to 90nm and smaller, process variations directly impact spiral inductor performance. The authors in [3] consider optimization of inductor parameters under process variability. Also, CMOS grade silicon has bulk conductivity which is quite high and variable [4]. Finally, when interconnect based structures like spirals are built on doped Silicon, the local doping density varies randomly and hence the conductivity also varies. Thus it becomes essential to model the substrate conductivity variations accurately since the Q is heavily impacted by substrate conductivity [5]. In addition, geometries are getting more and more prone to large variations as feature sizes get smaller due to lithography and DFM constraints.

The PMCHWT formulation decomposes the original problem into an equivalent set of interior and exterior problems. The scattering due to each object is now computed via the equivalent electric and magnetic surface current densities. This formulation gives us the advantage of analyzing finite sized dielectrics with a surface only based integral equation technique. The inductor is analyzed using an in-house code based on Multi-Region PMCHWT formulation developed at the ACE Lab.

2. Analysis method

For the On-chip inductor, two port y-parameters are computed and Y_{11} is used to compute the quantities of interest, the effective R, L and Q as in [4].

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$$R=\text{real}\left(\frac{1}{Y_{11}}\right),\quad L=\frac{\text{imag}\left(\frac{1}{Y_{11}}\right)}{2\pi f},\quad Q=-\frac{\text{imag}(Y_{11})}{\text{real}(Y_{11})} \tag{1}$$

Nine simulations are made for the ordered pairs of (σ_{si},w) per frequency point. Points in the interior region are approximated by a 2D Lagrange interpolation. If the values of a function f_{ij} (i=0 to M-1 and j=0 to N-1) are known on the grid points formed by the Cartesian product of two sets $\{x_0, x_1, x_{2,\ldots}x_{M-1}\}$ and $\{y_0, y_2, y_{2,\ldots}y_{N-1}\}$, then the interpolated value of the function at an interior point (x,y) is given by the well known expression

$$f_{xy}=\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left(\prod_{\substack{k=0\\k\neq i}}^{k=M-1}\prod_{\substack{l=0\\l\neq j}}^{l=N-1}\frac{(x-x_k)(y-y_l)}{(x_i-x_k)(y_j-y_l)}\right)f_{ij} \tag{2}$$

Here M=N=3. However, when the number of variables becomes large, this method becomes increasingly complex and regression techniques have to be resorted to.

Such an interpolation can be built for each of the parameters R, L and Q. A few intermediate points are simulated and the results are used to verify the interpolation. The next step is performing the Montecarlo analysis on the interpolated polynomial which saves time by a factor of 10^5 over a complete Montecarlo using the full-wave code (order of 10 simulations against 10^6 simulations). We consider 10^6 random vectors of (σ_{si},w) with each of the components having a normal distribution and the values of the parameters R,L,Q are found using (2). Once this is done, a simple, uniform binning process with sufficiently fine resolution can be used to find the PDFs of the requisite quantities.

3. Numerical results

A 3 turn square spiral inductor is designed to produce an inductance of 0.4 nH for an outer diameter of 60µm, trace width (w) of 5 µm, track spacing of 3 µm and metal thickness of 2 µm in a representative CMOS process. The conductivity is assumed to vary from 5 to 25 S/m. Nine simulations are done for the sets w={4,5,6} µm and σ_{Si} = {5,15,25} S/m. It is observed that L depends only on w and is linear for the considered range, while R and Q are affected by both variables and have non-linearities. Next, the mean of σ_{si} is as modeled as 15 S/m and standard deviation as 2 S/m and the mean of w is modeled to be 5µm and standard deviation as 0.2 µm. The PDF of R, L, Q is derived as outlined in section 2. Figures 1(a), 1(b) and 1(c) shows the PDFs of R, L and Q at 10 GHz and also provides a comparison with the Gaussian distribution with the same mean and variance. For R and Q, there is a slight deviation from the Gaussian profile due to the non-linear dependence while L follows the Gaussian profile very closely. Figure 2 shows how the PDF of Q varies when the variations in σ_{si} and w are changed. The Type 1 variation has standard deviations as 1S/m and 0.1µm while Type 2 variation has standard deviations 2 S/m and 0.2 µm. The standard deviations encountered decide interpolation ranges. Since extrapolated values obtained using the interpolated formulae are known to be inaccurate, it is expedient to model the interpolations from (μ-5σ) to (μ+5σ) so that almost no Montecarlo sample falls outside the interpolation range thereby generating accurate PDFs.

A simple yield simulation, based on a threshold of mean values for L(5% around the mean of 0.4 nH) and Q (greater than 10 at 10 GHz) is performed next. A simple post-processing step after the interpolation-based rapid Monte-Carlo analysis shows a 97%

yield for the Type 1 variation and a 73% yield for the Type 2 variation. Figure 3 shows the scatter plot of (L, Q) highlighting the optimal inductors for Type 2 variation.

4. Conclusions

This paper presents a simple and an effective way of obtaining the statistical properties of on-chip inductors affected by process variations. PDFs of various quantities of interest for on-chip spiral inductor design including R,L and Q where the random variables are the resistivity of the substrate and the width of metal traces, are obtained with a minimal number of full-wave simulations. Changes in PDF shape for different variability ranges of the input random variables were shown. Finally a simple yield prediction result was shown.

5. References

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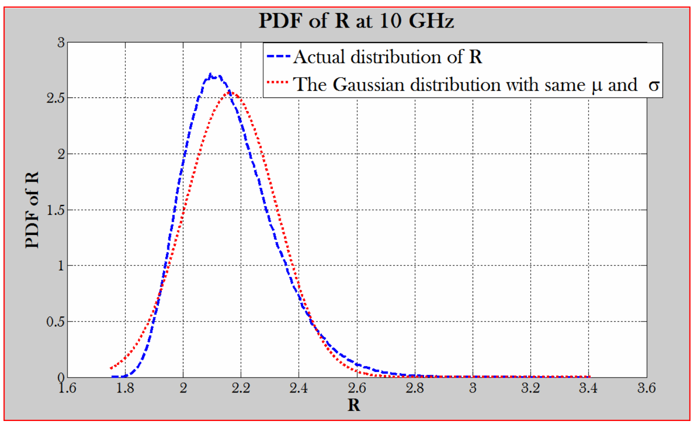

Figure 1a: PDF of effective R at 10 GHz

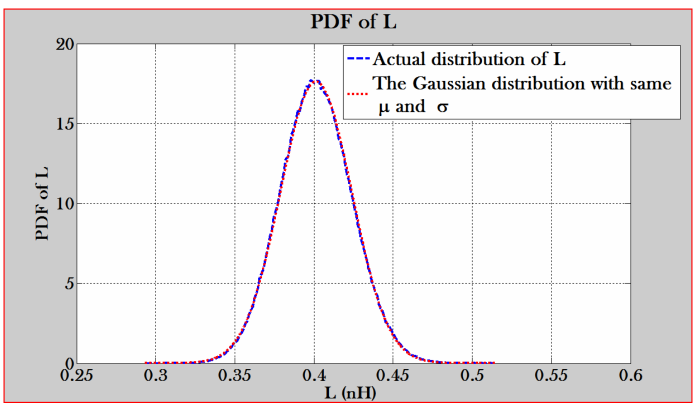

Figure 1b: PDF of L

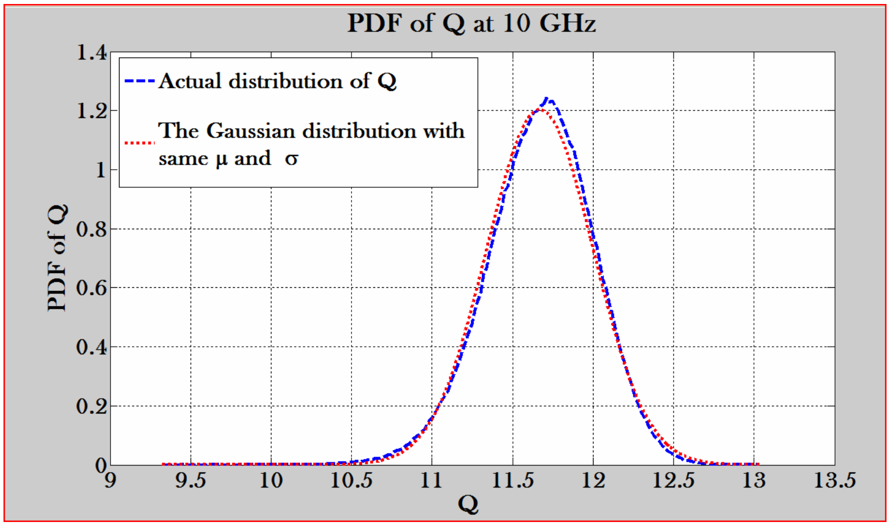

Figure 1c: PDF of Q at 10 GHz

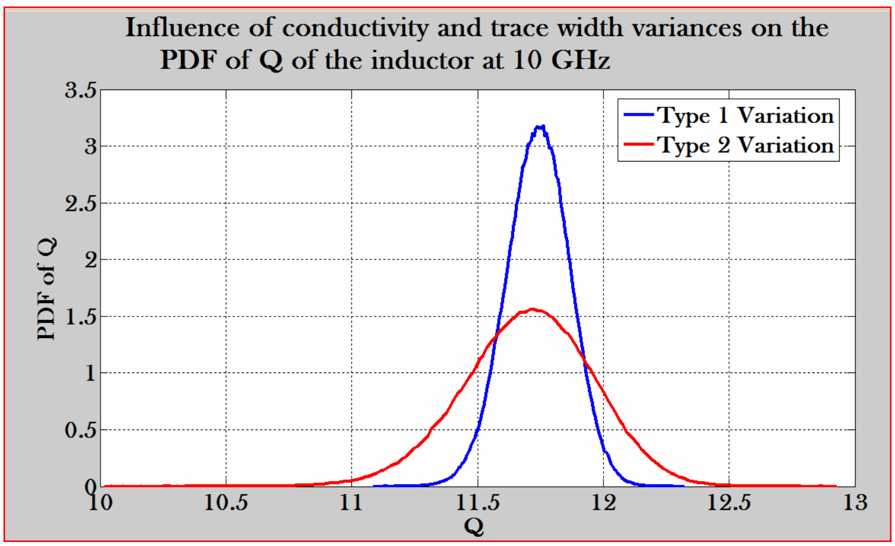

Figure 2: Illustrating the spreading of PDF for larger input parameter variations

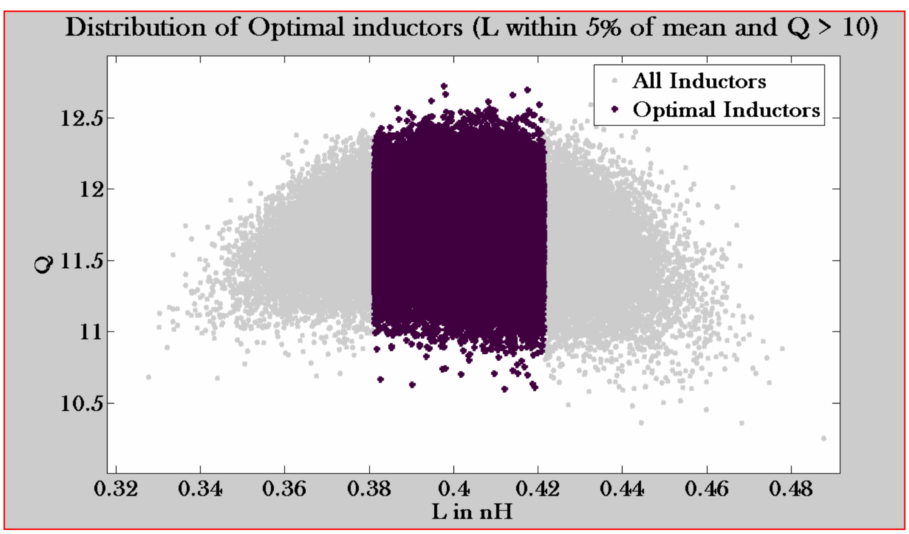

Figure 3: Distribution of optimal inductors in the L-Q space