A numerical solution of the layered media Green's function for the MPIE in the time domain

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1 Introduction

Layered media Green's functions in the frequency domain have been extensively studied because of important applications in geophysical probing and microstrip structures. Lately, the interest has increased in the evaluation of the layered media Green's functions in the time domain, to be subsequently used for time domain integral equations (TDIE's) [1]. This is because of critical time-domain applications including the analysis of transients in high speed digital interconnects and for analyzing wideband microstrip antennas. In using time domain layered Green's functions in conjuction with TDIE's, it is necessary to compute the layered medium Green's function for many source-to-field distances ρ and time intervals t.

In this paper, a numerical solution of the layered media (Figure 1) Green's function in the time domain for the mixed potential integral equation (MPIE) is reported. The particular case where the source and field points are on the top layer is presented. The method is applicable to multi-layered structures and to both lossless and lossy media. The technique also computes the Green's function for many ρ's and t's simultaneously. The salient features of the approach are as follows:

a) Complex frequency is employed to lift the poles off the real k_ρ axis. This alleviates the need to perform pole extraction [2]. Integration can be carried out on the real k_ρ axis [2] [3].
b) Half-space extraction [4] [5] is performed so that the integrand in the Sommerfeld integral for the non-half-space portion decays exponentially. The half-space portion is evaluated by branch cut integration.
c) The fast Hankel transform (FHT) [6]-[8] is used to evaluate the non-half-space portion for many ρ's simultaneously.
d) The fast Fourier transform (FFT) is used to transform the Green's function to the time domain for multiple t's.

2 Numerical solution for the mixed potential Green's functions for layered media

For a layered media with the source and the field on the top surface, the mixed potential Green's functions are given by

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$$G_A(\rho) = -j\frac{\mu}{4\pi}\int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho \rho)(1 + R^{TE})$$

$$G_V(\rho) = -\frac{j}{4\pi\varepsilon}\int_0^\infty dk_\rho \frac{1}{k_z}\left(k_\rho + \frac{k^2 R^{TE} + k_z^2 R^{TM}}{k_\rho}\right) J_0(k_\rho \rho)$$

where $\rho = \sqrt{x^2 + y^2}$ and R^{TE} and R^{TM} are the total reflection coefficients due to TE and TM polarized waves respectively incident on the top surface. For the rest of the paper, the solution methods for G_A will be shown. The same methods can be applied to G_V.

a) Transforming the Green's function to the time domain

The mixed potential Green's function may not be limited in frequency. It is thus necessary to multiply it (in the frequency domain) with a frequency-limited source function before the Fourier transform is carried out. Let the source function be of the Gaussian function:

$$x(t) = \frac{1}{\sqrt{\pi}\tau}\exp\left(-\frac{t - t_0}{\tau}\right)$$

The Fourier transform of $x(t)$ is

$$X(\omega) = \exp\left(-j\omega t_0 - \frac{\omega^2\tau^2}{4}\right)$$

Since complex frequency is used so that the poles are not on the real k_ρ axis, let $\omega = \omega' + j\omega''$. The time domain G_A is given as

$$G_A(\rho, t) = \frac{1}{\pi}e^{-\omega'' t}\,\mathrm{Re}\left(\int_0^\infty d\omega' e^{j\omega' t} X(\omega' + j\omega'') G_A(\rho, \omega' + j\omega'')\right)$$

b) Half-space extraction

The integrand of the Sommerfeld integral is slowly convergent when both the field and the source points are on the same interface (see figure 1, [4] [5]).

G_A can be expressed in terms of the half-space and non-half-space portions

$$G_A(\rho, \omega) = G_A^{(H)}(\rho, \omega) + G_A^{(N)}(\rho, \omega)$$

where

$$G_A^{(N)}(\rho, \omega) = -j\frac{\mu_0}{4\pi}\int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho \rho)(R^{TE} - R_{01}^{TE})$$

R_{01}^{TE} is the reflection coefficient of a TE polarized plane wave incident on the boundary between free space and the 1st layer only. $G_A^{(H)}$ can be evaluated by integrating along the vertical branch cuts at $k_\rho = k_0$ and $k_\rho = k_1$. After half space extraction, the integrand in the Sommerfeld integral decays exponentially.

c) Fast Hankel transform (FHT) solution for non-half-space portion

The basic idea of the FHT is to convert the Sommerfeld integral for $G_A^{(N)}(\rho,\omega)$ to a convolution integral that can be evaluated using the FFT. The details of the FHT can be found in [6]-[8].

d) Physical parameters

The 3 physical parameters of the problem are i) ω_{max}, the maximum angular frequency, ii) ρ_{max}, the maximum distance required and iii) $t_{largest}$, the largest time interval required. Let the width of the excitation pulse, $T_{width} = 2\pi/\omega_{max}$. $t_{largest}$ can be chosen as about 50 times of T_{width}.

3. Numerical results

Figure 2 shows $G_A(\rho,t)$ calculated for a multi-layered medium for the following characteristics: $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 4\varepsilon_0$, $\varepsilon_3 = 7\varepsilon_0$, $\varepsilon_4 = 11\varepsilon_0$. The thickness of the layers was 1.3 mm. A ground plane was below the 4^{th} layer as shown in Figure 1. The maximum frequency was 40 GHz. The imaginary part of the angular frequency, Im(ω), was chosen as $0.3/t_{largest}$. ρ_{max} was taken to be 10 cm. The ρ of interest was 3.75 cm. The values for comparison were calculated using real frequency with $G_A^{(H)}(\rho,\omega)$ evaluated on the Sommerfeld integration path (see Fig. 2 in [5]). The agreement between the two methods is excellent. Figure 3 shows $G_V(\rho,t)$ calculated for the same multi-layered medium. Again the agreement between the 2 methods is excellent.

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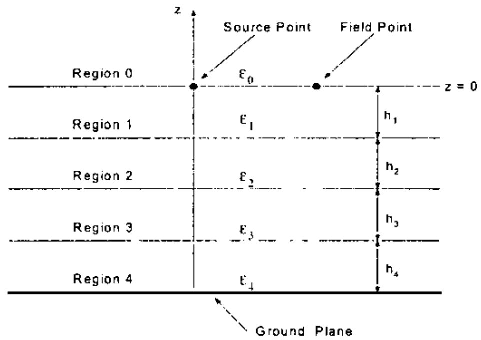

Figure 1 **A multi-layered medium. The source is at the origin**

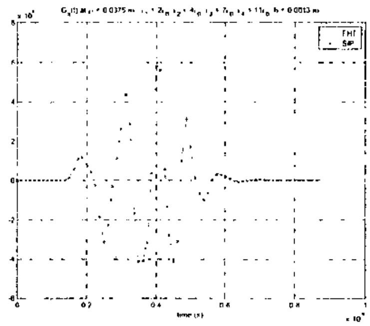

Figure 2 **Time domain vector potential Green's function for multi-layered medium**

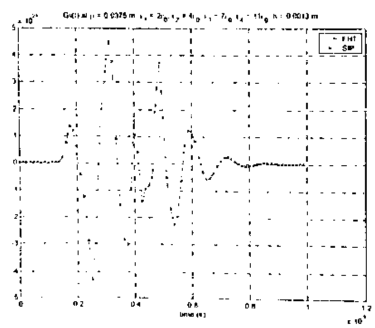

Figure 3 **Time domain scalar potential Green's function for multi-layered medium**