SDFMM-Based Fast Analysis of Radiation and Scattering from Finite Microstrip Structures

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1 Introduction

The prediction of radiation and scattering from microwave circuits and multilayered patch antennas is essential in the design of many communication systems. The analysis of electromagnetic phenomena involving large-scale quasi-planar structures requires efficient and accurate numerical techniques. The most popular approach for analyzing scattering and radiation from such structures relies on surface integral equation formulations and method of moments (MoM) based solution techniques.

The application of the MoM to solve surface integral equations leads to a matrix equation involving a dense matrix. In the past, the steepest descent fast multipole method (SDFMM) [1] has provided a means to reduce the CPU time and memory cost to $O(N)$ per iteration (versus the classical $O(N^2)$ cost) for iteratively solving the MoM equations for rough surface scattering. In this work, the SDFMM is applied to the full-wave analysis of microstrip structures on finite substrates and ground planes. The quasi-planar nature of such structures is exploited to obtain matrix-vector products in $O(N)$ CPU time and memory, resulting in dramatic solution efficiency. Owing to the multiregion nature of the present analysis problem, not all N MoM basis functions are independent. The required identification of independent basis functions, N_{ind} in number, and enforcement of appropriate boundary conditions are incorporated through another matrix transformation. The SDFMM permits the solution of scattering and radiation from extremely large and complex structures within realistic times. These include large microstrip patch arrays with finite substrates and ground planes modeled using 90,000 or more MoM basis functions.

2 Integral Equation Formulation for Scattering by Penetrable Microstrip Structures

To formulate integral equations for analyzing scattering and radiation from multi-region penetrable microstrip structures, we follow the approach outlined in Ref.[2]. An arbitrary number of homogeneous regions R_T is permitted. The interface between any two regions can be a penetrable boundary, a perfectly conducting boundary, or a segmented combination. The overall CFIE can be obtained by enforcing constraints on the fields and currents. Specifically, one equates tangential components of total electric and magnetic fields across

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a penetrable interface, and enforces zero total electric field on a perfectly electrically conducting (PEC) boundary. An independent set of currents is obtained by introducing the constraints that equivalent currents are equal and opposite across a penetrable interface, and that magnetic currents are zero on a PEC surface. On enforcing these constraints and using the MoM, we arrive at the following full-rank matrix equation [2, 3]

$$\bar{\mathbf{A}}\bar{\mathbf{Z}}\bar{\mathbf{A}}^T \mathbf{I}_{ind} = \bar{\mathbf{A}}\mathbf{V} , \tag{1}$$

where $\bar{\mathbf{Z}}$ is the dense MoM matrix, $\bar{\mathbf{A}}$ is a sparse matrix that enforces the above constraints, $\mathbf{I}_{ind}$ is the set of coefficients of independent basis functions, and $\mathbf{V}$ is the excitation vector which could be obtained from a plane-wave, a wire excitation, or a coaxial probe modeled as an excitation from a surface-wire junction. The solution of the constrained CFIE (Eqn.(1)) yields the electric and magnetic surface current densities. Once the current densities have been obtained, radar cross sections (RCSs) or radiation patterns can be computed.

3 The Steepest Descent–Fast Multipole Method

The MoM matrix $\bar{\mathbf{Z}}$ in Eqn.(1) possesses dimensions of $N \times N$, where N is the total number of basis functions, and has the form

$$\bar{\mathbf{Z}} = \begin{bmatrix} \bar{\mathbf{L}}_1 & -\bar{\mathbf{K}}_1 & \mathbf{0} & \mathbf{0} & \dots \\ \bar{\mathbf{K}}_1 & \frac{\epsilon_1}{\mu_1}\bar{\mathbf{L}}_1 & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{0} & \bar{\mathbf{L}}_2 & -\bar{\mathbf{K}}_2 & \dots \\ \mathbf{0} & \mathbf{0} & \bar{\mathbf{K}}_2 & \frac{\epsilon_2}{\mu_2}\bar{\mathbf{L}}_2 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} , \tag{2a}$$

with the entries of the submatrices given by

$$\bar{\mathbf{L}}_q(m,n) = \langle \mathbf{f}_{q,m}, L_q\, \mathbf{j}_{q,n} \rangle \ , 1 \le m,n \le N_q, \tag{2b}$$

$$\bar{\mathbf{K}}_q(m,n) = \langle \mathbf{f}_{q,m}, K_q\, \mathbf{j}_{q,n} \rangle \ , \ 1 \le m,n \le N_q \ , \tag{2c}$$

where $\mathbf{j}_{q,n}$ and $\mathbf{f}_{q,m}$ are basis and testing functions, L_q and K_q are Green's function operators [2], and N_q is the number of basis/testing functions in region q, with $\sum_{q=1}^{R_T} N_q = N$. An iterative solution of Eqn.(1) is expensive, since both the CPU time per iteration and the memory scale as $O(N^2)$. For large scale problems, one needs to develop efficient fast algorithms to alleviate the computational burden. To this end, the SDFMM [1] is introduced. In a single level implementation of the multi-region SDFMM, the microstrip structure is embedded in a block, which is then subdivided into smaller blocks in the x- and y- directions. A matrix element is classified as a near-field element if the corresponding basis and testing functions reside in blocks which are separated by less than a pre-specified number of blocks. All other elements are termed far-field elements. This classification is used to formally decompose the impedance matrix $\bar{\mathbf{Z}}$ as

$$\bar{\mathbf{Z}} = \bar{\mathbf{Z}}' + \bar{\mathbf{Z}}'' \tag{3}$$

where $\bar{\mathbf{Z}}'$ and $\bar{\mathbf{Z}}''$ contain near- and far-field interactions, respectively.

In the SDFMM, the action of $\bar{\mathbf{Z}}'$ on a vector is computed classically. However, the product of $\bar{\mathbf{Z}}''$ with a vector is computed indirectly and rapidly, without ever generating the matrix. This procedure is briefly summarized here. The three-dimensional dynamic scalar Green's function is first expressed in a contour integral form, using the Sommerfeld identity. This integral can be evaluated efficiently through a steepest descent path integration. Moreover, a Hankel function appearing in the integrand is expanded in the spectral domain using the addition theorem. This overall steepest-descent fast multipole expansion can be used to efficiently represent terms arising in the product of $\bar{\mathbf{Z}}''$ with a trial vector in the following manner [1]

$$\langle \mathbf{f}_{q,m}, L_q\, \mathbf{j}_{q,n} \rangle = \frac{i}{16\pi^2} \sum_{j=1}^{n_{sd,q}} \sum_{j'=1}^{P_q} w_{jq}^{sd} w_{j'q}^{fmm}\, k_{\rho q}^{(j)} \int_S d\mathbf{r}\, \mathbf{f}_{q,m} e^{i\mathbf{k}_q^{(j)}\cdot(\mathbf{r}-\mathbf{r}_t)} \quad (4)$$
$$\mathcal{T}_{jj'q}(\mathbf{r}_t - \mathbf{r}_s) \left(\bar{\mathbf{I}} - \frac{\mathbf{k}_q^{(j)}\mathbf{k}_q^{(j)}}{k_q^2} \right) \int_S d\mathbf{r}'\, \mathbf{j}_{q,n} e^{i\mathbf{k}_q^{(j)}\cdot(\mathbf{r}_s-\mathbf{r}')} ,$$

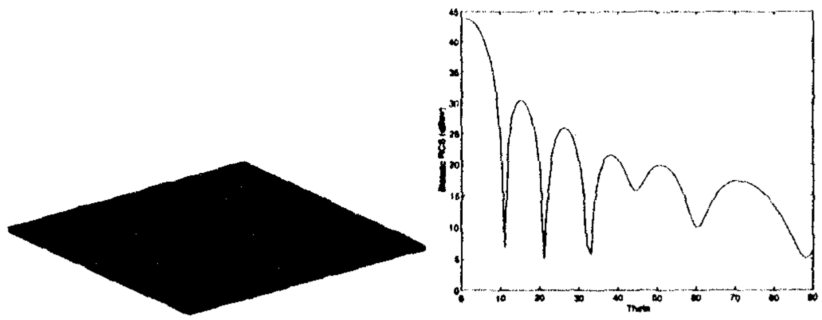
and

$$\langle \mathbf{f}_{q,m}, K_q\, \mathbf{j}_{q,n} \rangle = -\frac{1}{16\pi^2} \sum_{j=1}^{n_{sd,q}} \sum_{j'=1}^{P_q} w_{jq}^{sd} w_{j'q}^{fmm}\, k_{\rho q}^{(j)} \int_S d\mathbf{r}\, (\mathbf{f}_{q,m} \times \mathbf{k}_q^{(j)}) e^{i\mathbf{k}_q^{(j)}\cdot(\mathbf{r}-\mathbf{r}_t)} \quad (5)$$
$$\mathcal{T}_{jj'q}(\mathbf{r}_t - \mathbf{r}_s) \int_S d\mathbf{r}'\, \mathbf{j}_{q,n} e^{i\mathbf{k}_q^{(j)}\cdot(\mathbf{r}_s-\mathbf{r}')} .$$

Here, $\mathbf{k}_q^{(j)}$ are complex wavenumbers,and $n_{sd,q}$ and $w_j^{sd,q}$ are the number of points and integration weights associated with integration along the steepest descent path of the Sommerfeld integral representation of the free-space Green's function [1]. Source and testing block centers are denoted by $\mathbf{r}_s$ and $\mathbf{r}_t$, respectively. The translation operator $\mathcal{T}_{jj'q}(\mathbf{r}_t - \mathbf{r}_s)$, integration weights $w_{j',q}^{fmm}$, and number of harmonics P_q are analogous to those defined in well-known fast multipole algorithms. In a multilevel SDFMM, the microstrip structure is recursively divided into blocks, by hierarchically partitioning a block (the *parent*) at a coarse level into four blocks (the *children*) at a finer level. Plane-wave expansions are shifted to centers of parent blocks, and incoming spectra are shifted to centers of child blocks. Such an operation is termed an FMM tree traversal. Distinct translation operators and steepest descent rules are utilized at each FMM level. For an R_T region problem, R_T separate FMM trees and tree-traversals are necessitated. Moreover, it can be shown that the effect of both the electric and the magnetic surface currents in a particular region can be accounted for through a single tree formation and traversal. Also, magnetic fields at the observer locations are obtained through transforming each plane-wave component of the computed electric field.

4 Simulation Results

In this section, numerical results are presented which demonstrate the efficacy of the SDFMM in analyzing scattering and radiation from large microstrip structures. The computing platform used is a single R8000 processor on an SGI Power Challenge, with 2 Gbytes of RAM and an average in-program throughput of 60 MFlops. A TFQMR iterative solver is used with the SDFMM. The multi-region SDFMM is applied to analyze scattering from a large microstrip patch array (Fig. 1(a)). The overall dimensions of the finite substrate ($\epsilon_r = 2.17$) are $6.1\lambda \times 6.1\lambda \times 0.05\lambda$. The 10×10 microstrip array, the substrate, and the finite ground plane below the substrate are modeled using a total of $N = 92,280$ MoM basis functions, with $N_{ind} = 41,952$. The bistatic RCS for a normally incident plane wave is depicted in Fig. 1(b)). The second example involves a 6×6 probe-excited array. The patches have dimensions $0.3\lambda \times 0.3\lambda$, and are separated by 0.2λ in each direction. The substrate ($\epsilon_r = 2.55$) possesses dimensions of $3.2\lambda \times 3.2\lambda \times 0.04\lambda$. The normalized radiation pattern for a balanced in-phase excitation is shown in Fig. 2. Coaxial probes are modeled using 36 wires, each having 5 nodes, with junctions connecting the wire to the ground plane and patch, and excitation provided at the lower junctions.



(a) (b)

Figure 1: (a) 10 × 10 microstrip array with a finite substrate and ground plane. (b) Bistatic RCS for a normally incident plane wave.

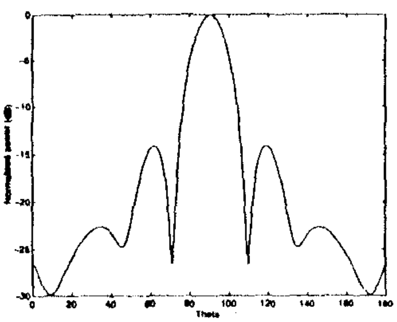


Figure 2: Normalized radiation pattern of a probe-excited 6 × 6 array

5 Conclusions

A new multilevel algorithm, the SDFMM, has been shown to permit the rapid analysis of scattering and radiation from microstrip structures. Due to the dramatic speedup and memory savings possible, it is expected that the SDFMM will become a useful tool in microwave circuit analysis and design.

References

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