

A Combined Steepest Descent-Fast Multipole Algorithm for the Analysis of Three-Dimensional Scattering by Rough Surfaces

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Introduction

The scattering of electromagnetic waves by random rough surfaces is a subject of great practical and intellectual interest. Applications [1] include, but are not limited to, remote sensing, long-range communications, radio astronomy, medicine, and surface physics. Random surfaces also exhibit several interesting scattering and polarization characteristics, such as backscattering enhancement and the generation of localized modes.

A host of numerical techniques exist for analyzing rough surfaces; see Ref. [2] and references therein. However, it is only recently that the fast algorithms have been developed, enabling analysis of larger surfaces. In this paper, an efficient algorithm of $O(N)$ computational complexity will be presented. The salient features of this approach are:

- Advantage is taken of the fact that a rough surface is *nearly* planar to derive efficient numerical integration rules for the Sommerfeld integral representation of the free space Green's function.
- The Hankel functions arising in such an integration are evaluated using a Fast Multipole Method (FMM)-like algorithm that is tailored towards rough surfaces.
- The proposed algorithm, called the Steepest Descent-Fast Multipole Method (SDFMM), has $O(N)$ CPU time *and* storage requirements unlike all other previously proposed methods. The technique is numerically rigorous and its accuracy can be controlled as desired.

Formulation

The crux of the SDFMM lies in an efficient evaluation of the Green's function. Consider the following integral representation of the scalar Green's function

$$g(\mathbf{r}, \mathbf{r}') = \frac{e^{ik_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} = \frac{i}{8\pi} \int_{-\infty}^{\infty} dk_z e^{ik_z(z-z')} H_0^{(1)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|). \quad (1)$$

Transforming the above equation using $k_z = k_0 \sin \alpha$ and $k_\rho = k_0 \cos \alpha$, together with the large argument approximation of the Hankel function, yields

$$\frac{e^{ik_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \cong \frac{i}{8\pi} \sqrt{\frac{2k_0}{\pi|\boldsymbol{\rho}-\boldsymbol{\rho}'|}} e^{-i\pi/4} \int_{-\infty}^{\infty} d\alpha \sqrt{\cos \alpha} e^{ik_0((z-z') \sin \alpha + |\boldsymbol{\rho}-\boldsymbol{\rho}'| \cos \alpha)}. \quad (2)$$

The saddle point of the integral appearing in Eqn. (2) occurs at the origin when $z - z'$ is zero. Also, the condition of constant phase yields

$$\cos \alpha_R \cosh \alpha_I = 1 \quad (3)$$

for $\alpha = \alpha_R + i\alpha_I$. The integral in Eqn. (1) can now be numerically along the Steepest Descent (SD) path. For small α_R , Eqn. (3) reduces to $\alpha_R = -\alpha_I$, and the integrand decays exponentially on either side of $\alpha_R = (z - z')/|\rho - \rho'|$, while the phase variation is governed only by $(z - z')$. These characteristics of the integrand allow the development of an efficient and robust numerical rule for which only the extent of the integration domain and the number of points have to be determined. The former depends on ρ while the latter depends on the phase variation. Thus, larger z would require more points in the rule. But in a rough surface, as the z variations are small compared to the lateral dimensions. Hence, robust numerical integration can be performed with a small number of points.

Thus, the scalar Green's function can be expressed as

$$g(\mathbf{r}, \mathbf{r}') \cong \frac{i}{8\pi} \sum_{j=1}^{n_{sd}} w_j k_\rho^{(j)} e^{ik_z^{(j)}(z-z')} H_0^{(1)}(k_\rho^{(j)}|\rho - \rho'|) \quad (4)$$

with $k_\rho^{(j)} = k_0 \cos \alpha_j$, $k_z^{(j)} = k_0 \sin \alpha_j$ and α_j and w_j represent the j^{th} integration point along the SD path and its corresponding weight. To implement the FMM, the rough surface is divided hierarchically into blocks lying on a plane, by recursively dividing each block at a particular level into four smaller blocks at the finer level.

At the finest level, sources lying in each block are represented by plane wave expansions located at the center of each block, irrespective of the particular z location of the source. The Hankel function can then be expressed as:

$$H_0^{(1)}(k_\rho^{(j)}|\rho - \rho'|) \cong \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{ik_\rho^{(j)}(\rho - \rho_c) \cdot \hat{s}} \mathcal{T}(k_\rho^{(j)}, \hat{s}, \rho_c - \rho_a) e^{ik_\rho^{(j)}(\rho_a - \rho') \cdot \hat{s}}, \quad (5a)$$

$$\mathcal{T}(k_\rho^{(j)}, \hat{s}, \rho_c - \rho_a) = \sum_{p=-P}^P H_p^{(1)}(k_\rho^{(j)}|\rho_c - \rho_a|) e^{-ip[\theta - \phi - \pi/2]}, \quad (5b)$$

where the integral represents contributions from the entire plane wave spectrum, ρ_a and ρ_c are FMM box centers, and \mathcal{T} is the translation operator which depends on the complex wavenumber, spectral angle, and the displacement between source and observation boxes. Also, $\hat{s} = \hat{x} \cos \phi + \hat{y} \sin \phi$, and $\cos \theta = \hat{x} \cdot (\rho_c - \rho_a)/|\rho_c - \rho_a|$. Using Eqns. (4) and (5), the dyadic Green's function $\tilde{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$ can now be written as

$$\tilde{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cong \frac{i}{16\pi^2} \sum_{j=1}^{n_{sd}} w_j e^{ik_z^{(j)}(z-z')} k_\rho^{(j)} \left(\bar{\mathbf{I}} - \frac{\mathbf{k}^{(j)} \mathbf{k}^{(j)}}{k_0^2} \right) \times \quad (6a)$$

$$\int_0^{2\pi} d\phi e^{ik_\rho^{(j)}(\rho - \rho_c) \cdot \hat{s}} \mathcal{T}(k_\rho^{(j)}, \hat{s}, \rho_c - \rho_a) e^{ik_\rho^{(j)}(\rho_a - \rho') \cdot \hat{s}} \quad (6b)$$

$$\mathbf{k}^{(j)} = k_\rho^{(j)} \hat{\rho} + k_z^{(j)} \hat{z}.$$

The SDFMM implementation further uses a multilevel version [3] of the FMM described above.

Results

The SDFMM described in the previous section is implemented together with a windowed translation operator which allows for further reduction in computational complexity. It

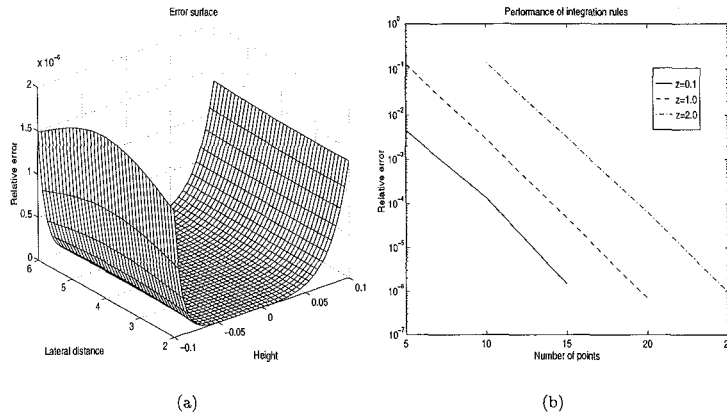


Figure 1: Performance of integration rules

should be noted that the SDP integration must be valid for all source observation displacements in the interaction range. This implies that a *fixed* set of integration weights w_j and locations $k_p^{(j)}$ (or equivalently $k_z^{(j)}$) have to be valid for a *range* of z and ρ . The accuracy of these integration rules together with their dependence on z is shown in Figs. 1(a,b). This algorithm is first validated against the standard Method of Moments (MoM) technique for a rough surface of $3.9\lambda \times 3.9\lambda$ with roughness $\sigma = 0.5\lambda$ and correlation length $l = 1.5\lambda$, and as seen in Fig. 2(a) the agreement is excellent. Furthermore, the computational complexity of this algorithm is of $O(N)$ (Fig. 2(b)). Finally, a Monte-Carlo simulation was carried out for 50 rough surfaces of dimension $5.9\lambda \times 5.9\lambda$ with $\sigma = 0.5\lambda$, $l = 1.5\lambda$ and the total number of unknowns being 10,325. The results of the non-coherent bistatic scattering coefficients for co- and cross-polarized components are depicted in Figs. 3(a,b).

Acknowledgments: This work was supported in part by a grant from AFOSR via the MURI Program under contract no F49620-96-1-0025.

References

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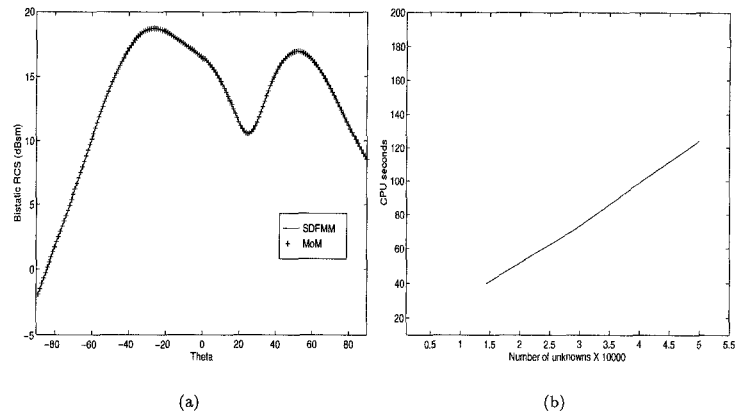


Figure 2: (a) Comparison between SDFMM and MoM (b) CPU time for matrix-vector multiplication

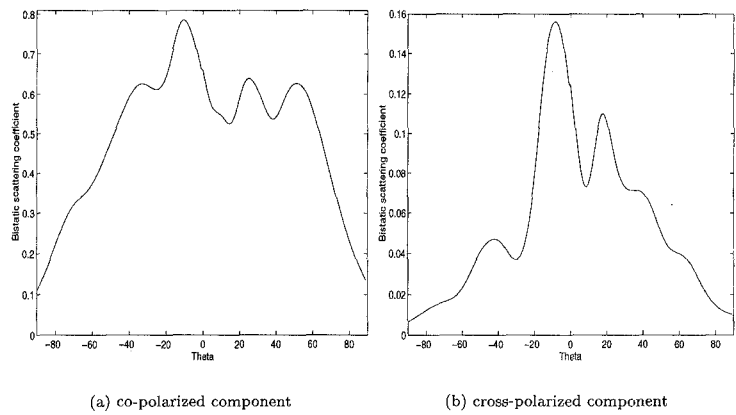


Figure 3: Monte Carlo simulation