Fast Capacitance Computations for Conducting Structures Embedded in a Multilayered Dielectric Medium Using the Fast Multipole Algorithm and a Closed Form Green's Function

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Abstract

Efficient computation of the capacitance matrices of arbitrarily-shaped three-dimensional conducting structures embedded in a multilayered dielectric medium is discussed in this paper. It is shown that the fast multipole method (FMM) employed in conjunction with an iterative method and a closed-form approximation of the Green's function can speedup the computation time by about an order of magnitude over conventional direct and iterative methods without compromising the numerical accuracy of the results.

0-7803-2411-0/94/$4.00 ©1994 IEEE

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Summary

Accurate and efficient computation of capacitance matrices of three-dimensional multiconductor structures is extremely vital for predicting the signal integrity, ensuring the proper functioning of electronic packages and integrated-circuit interconnects, and reducing the design cycle time. Typically, the conductors are embedded in multilayered dielectric media, and this places heavy demands on the memory and cpu time required for capacitance computation.

Nabors *et al.* have developed an efficient algorithm based on the Fast Multipole Method (FMM), applied in conjunction with the Method of Moments (MoM) and an iterative solution procedure, for capacitance computation in a uniform dielectric [1,2] and in the presence of multiple, arbitrarily-shaped, finite dielectrics [3,4]. The iterative techniques they have used include the conjugate gradient method [5] and the Generalized Minimal Residual (GMRES) algorithm [6].

In this work, we address the problem of adapting the FMM approach for computing the capacitance of arbitrarily-shaped three-dimensional conductors placed in a stratified dielectric medium, by using a closed-form Green's function for such a medium. In a recent paper Chow *et al.* [7] have derived a closed-form approximation to the Green's function for stratified dielectric media by utilizing a finite number of complex images, and Oh *et al.* [8] have accomplished the same with only real images. In our work, the latter type of images are incorporated into the FMM formulation proposed by Anderson [9], which is then employed in conjunction with the GMRES [6] algorithm to solve the capacitance problem. It is shown that this technique can be used to analyze a variety of conducting structures embedded in stratified dielectric media, and that it is much faster than conventional direct or iterative methods that do not utilize the FMM.

To illustrate the application of the method, we consider the problem of estimating the capacitance of a conducting plate (Fig. 1), and that of an air bridge (Fig. 2), both of which are assumed to be embedded in a dielectric layer with a thickness of 5×10^{-3} m and a relative permittivity of 5, and which is backed by a ground plane underneath. The performance of the algorithm presented herein, which solves the MoM equations by employing the FMM in conjunction with the GMRES iterative solver (referred to henceforth as FMM-GMRES) is compared with those of two non-FMM, MoM-based solvers. The latter two rely on LU decomposition and on the GMRES iterative method, and are referred to herein as NFMM-LU and NFMM-GMRES, respectively. All computations have been performed on a SUN SPARC-10 workstation and the cpu times required by these three techniques are shown in Figs. 3(a) and 3(b). It is evident that the FMM-GMRES is not only considerably faster than the other two methods, it also exhibits a behavior of computational complexity which is linear in nature. The capacitance matrix entries obtained by using the FMM-GMRES technique are found to be accurate to within one per cent of those derived by using the non-FMM methods.

Some refinements of the present algorithm, described in [10], are currently being implemented in a computer program, and are expected to lead to further savings in the cpu time and memory.

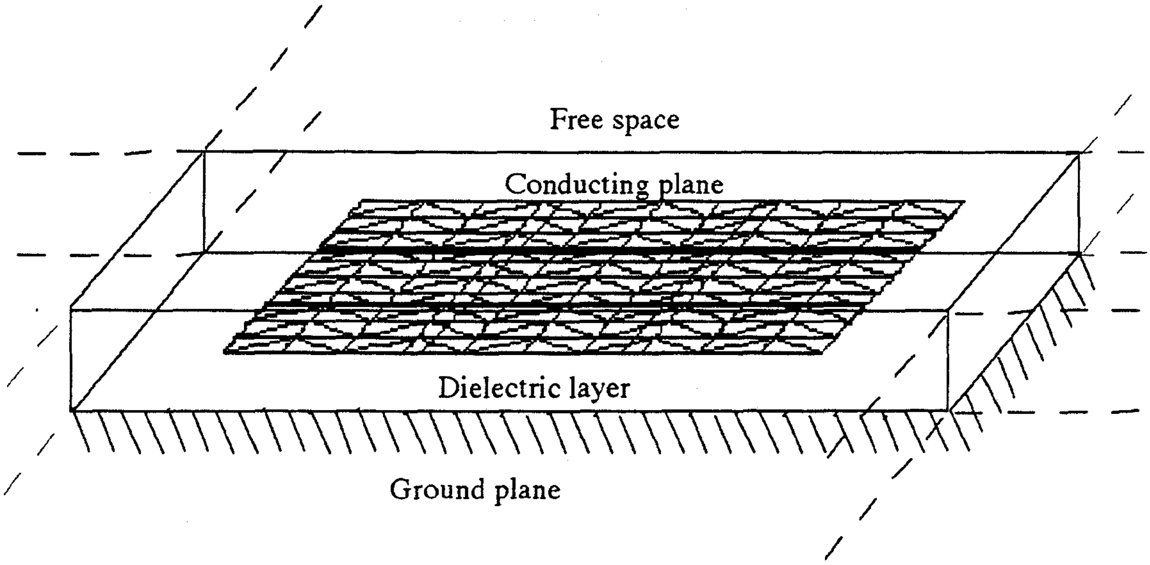


Figure 1 : Conducting plate embedded in a dielectric layer. The dielectric layer has a relative permeability of 5 and a thickness of 5×10^{-3} m. The plate is placed in the center of the dielectric.

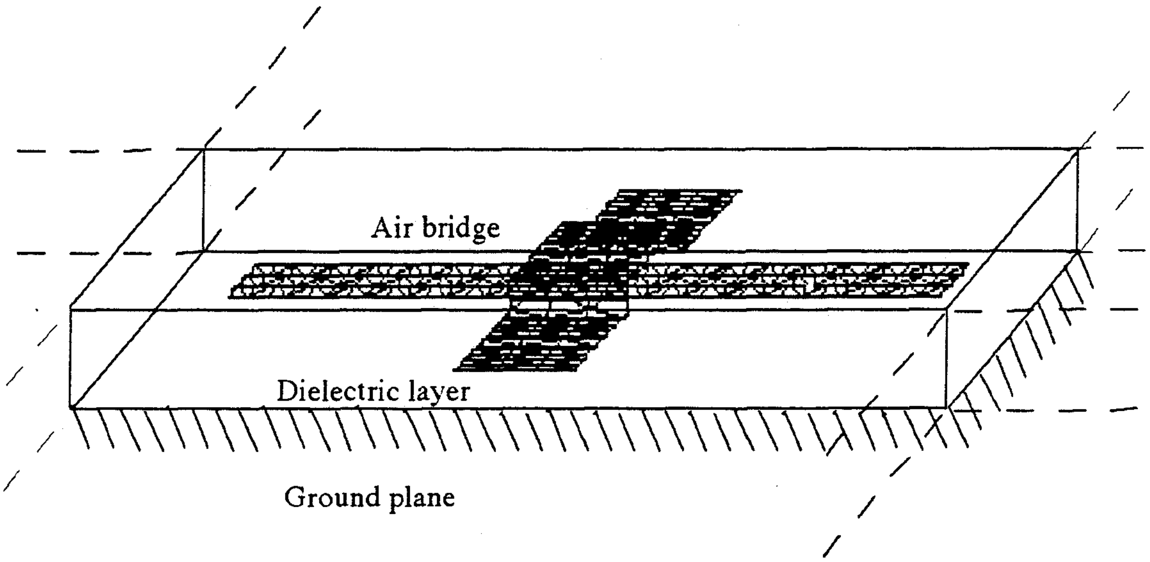
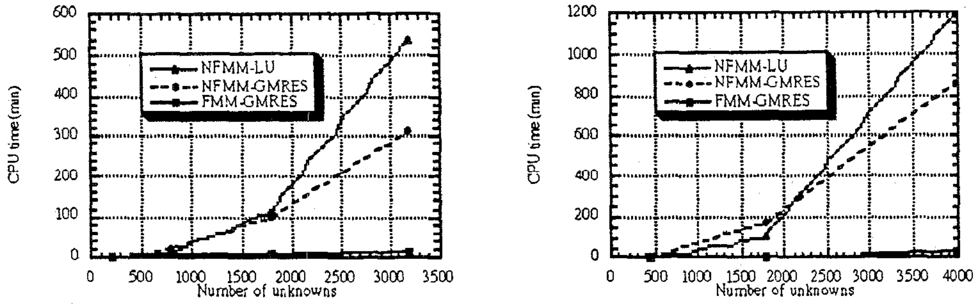


Figure 2 : Air bridge embedded in a dielectric layer. The dielectric has a relative permeability of 5 and a thickness of 5×10^{-3} m.



(a) *(b)*

Figure 3 : CPU time required for obtaining the capacitances using NFMM-LU, NFMM-GMRES, and FMM-GMRES versus number of unknowns for the conducting plate example (Fig. 3(a)), and for the air bridge example (Fig. 3(b)).

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