# Coupled Electromagnetic-Circuit Simulation of Arbitrarily-Shaped Conducting Structures using Triangular Meshes

Vikram Jandhyala, Yong Wang, Dipanjan Gope, and Richard Shi

Department of Electrical Engineering, University of Washington, Seattle, WA 98195

Ph: 206-543-2186 Fax: 206-543-3842, email {jandhyala, cjshi}@ee.washington.edu

## Abstract

***The Partial-Element-Equivalent-Circuit (PEEC) approach is an effective method to convert three-dimensional on-chip multi-conductor structures to circuit-level descriptions. In this paper, a triangular-mesh-based PEEC approach is described, wherein the surfaces of arbitrarily-shaped conducting structures are represented by triangular mesh tesselations. A coupled EM-circuit formulation is obtained through the separation of the scalar, vector, and ohmic potential interactions between pairs of triangular edges-based basis functions. The overall approach can be interpreted as a SPICE-free, surface-only version of PEEC method and is especially useful for on-chip signal integrity analysis of systems-on-chip layout where components with irregular shapes are common.***

## I. Introduction

Recently, Systems-on-Chip (SoCs) have become one of the focus areas in VLSI. Through the integration of analog and digital parts into a single chip, the resulting system under design achieves more reliability and shorter manufacture to market cycle than solutions based on individual digital and analog integrated circuits. Meanwhile, new problems have emerged in SoC design [1], e.g., the fast-switching current in the digital part of a chip can electromagnetically couple to the analog part which is noise sensitive. This is especially true when the system is functioning at the GHz range or beyond.

In order to analyze crosstalk due to EM coupling, electromagnetic simulation is needed for the layout of the SoC chip. The PEEC method [2] is a particularly effective approach to model the electromagnetic effects of a multi-wire or multi-conductor structure using SPICE compatible elements.

The interaction of a multi-wire or multi-conductor structure can be described using the Electric Field Integral Equation (EFIE) [3]. In classical electromagnetics (EM), the EFIE is usually formulated using a Method of Moments (MoM) approach [4]. Instead of filling the Method of Moments (MoM) matrix and solving the resultant set of linear equations, the PEEC method extracts partial elements including resistance, self/mutual capacitance and self/mutual inductance from the EFIE formulation, by identifying these elements with ohmic, scalar, and vector potential interactions, respectively. A SPICE compatible netlist can then be generated using these extracted partial elements. Through this extraction, the original EM problem is converted to a circuit problem, and a circuit simulator can then predict the performance of a layout while automatically considering the electromagnetic effects due to geometry and structure [2].

The classical PEEC method, originally formulated for modeling crosstalk between digital traces, relies on a longitudinal filament discretization of all structures. This discretization, which assumes a direction of current flow along the length of the filament, is very well-suited for thin and long interconnect structures. However, an SoC scenario leads to several arbitrarily shaped structures, including spiral inductors, and regular and split ground planes wherein the filament approach is inherently not well suited, because of the arbitrary directions of current flow in such structures. Moreover, it is not intuitive to represent current flow on non-longitudinal structures in terms of scalar longitudinal filaments.

In this paper, Rao-Wilton-Glisson (RWG) basis functions [4,5] that are linear basis functions defined over triangles are used to model conductors using surface-only triangular meshes. Interactions between RWG basis functions are then extracted and were used to form a coupled matrix which also includes MNA formulation of circuits. The aim of developing a coupled formulation outside SPICE is to reduce reliance on sparse-matrix solvers, since sections of the coupled matrix system are inherently dense.

The rest of the paper is organized as follows. Section II briefly outlines the classical filamental PEEC method, and introduces the RWG triangle-basis functions. Section III presents the triangular mesh approach and the coupled EM-circuit formulation. Numerical simulation results are given in Section IV and Second V summarizes the paper.

## II. Mesh Generation: Filaments versus Triangle Meshes

The PEEC method, originally developed to model digital interconnects, inherently assumes that filaments, i.e. thin and long structures, can be used to model sections of the structure under analysis, as well as the current flow (along the filament length). As shown in Fig. 1, filamental PEEC divides the object into filaments. Each filament, represented as a volume

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cell, represents a longitudinal current, and related surface cells represent surface charge.

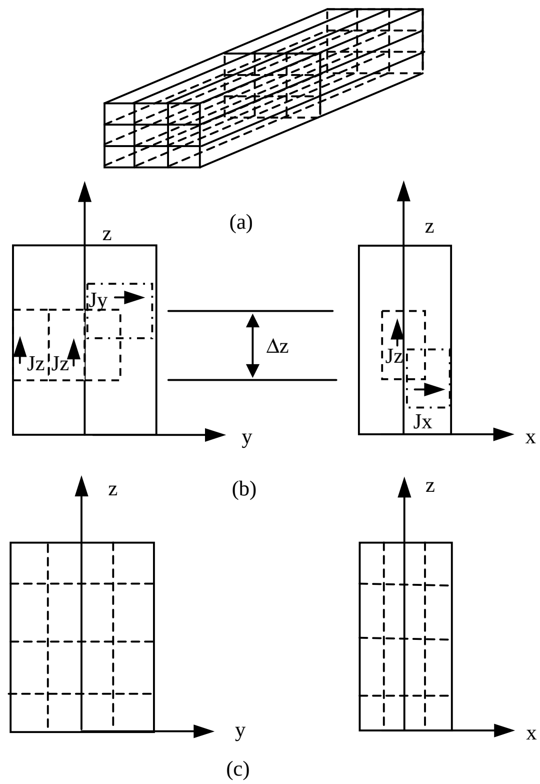


**Figure 1. (a) rectangular conductor divided into filaments (b) Volume cells for currents (c) Surface cells for capacitive**

For structures where 2D (e.g. a thin ground plane) or 3D (e.g. a thick ground plane) current distribution is necessitated, independent discretizations in terms of filaments in each direction are required, and the efficiency of the filamental PEEC method rapidly degrades. Also inherent in filamental approach is an eventual staircase approximation to the current distribution. In general, filamental PEEC is ideal for long rectangular structures under the assumption that the currents flow only along the longitudinal direction.

In this paper, triangular meshes are used to represent arbitrarily-shaped surfaces. Common edges between triangles are used to define RWG basis functions that define current flow and charge distribution. Fig. 2 shows the triangle pair on which the current density and charge density is defined. An RWG basis function, defined with respect to the common edge, defines current flows from one triangle (+) across the common edge to the other (-) triangle.

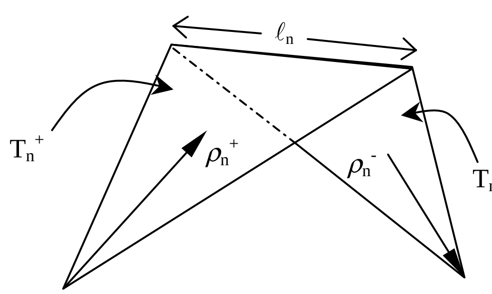


**Figure 2. Definition of triangle basis function**

## III. Coupled EM-Circuit System Formulation

In the MoM, conducting structures are analyzed using the electric field integral equation formulation (EFIE), wherein the surface current density $\boldsymbol{J}$ satisfies the equation:

$$j\omega \frac{\mu}{4\pi} \int_S \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' + (\nabla \phi)(\mathbf{r}) = -Z_s \mathbf{J}(\mathbf{r}) \qquad (3.1)$$

Scalar potential $\phi$ and surface charge density $\rho$ are related through the equation:

$$\phi = \frac{1}{4\pi\varepsilon} \int_S \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \qquad (3.2)$$

In the above equation $Z_s$ represents surface impedance

$$Z_s = (1+j)\sqrt{\frac{\omega\mu}{2\sigma_v}} \qquad (3.3)$$

which is a valid approximation at frequencies where the skin depth is smaller than the cross section of conductors. At lower frequencies, a second (interior) problem and accurate modeling of the lossy medium Green function within the conductor is required.

Upon testing the EFIE, the following matrix equation can be derived, entries of the matrix can then be extracted from the interaction of basis function.

$$j\omega \overline{\mathbf{L}}\mathbf{J} + \overline{\mathbf{A}}\mathbf{V} = -\overline{\mathbf{Z}}\mathbf{J} \qquad (3.4)$$

$$\overline{\mathbf{P}}\,\mathbf{Q} = \mathbf{V} \qquad (3.5)$$

After dividing the surface of the object into triangular meshes, the unknowns of interest are: distribution of scalar potential $\phi$, surface charge density q, and surface current density $\boldsymbol{J}$. These quantities are expanded using basis functions defined over triangles:

$\phi$: piecewise constant basis function $\phi = \sum_{n=1}^{Np} V_n C_n$

*Np* is the number of total patches, $C_n$ is a piecewise constant basis function, which is 1 on triangle *n*, and 0 elsewhere.

q: piecewise constant basis function $q = \sum_{n=1}^{Np} Q_n C_n$

$\boldsymbol{J}$ : RWG basis function $\boldsymbol{J} = \sum_{n=1}^{Ne} J_n \mathbf{f}_n(\mathbf{r})$

*Ne* is the number of total non-boundary edges, $\mathbf{f}_n$ is RWG basis function that is defined as:

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \frac{l_n}{2A_n^+}\rho_n^+ & \text{when } \mathbf{r} \text{ is in } T_n^+ \\ \frac{l_n}{2A_n^-}\rho_n^- & \text{when } \mathbf{r} \text{ is in } T_n^- \\ 0 & \text{elsewhere} \end{cases} \qquad (3.6)$$

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In the above expression, $l_n$ is the length of the common edge, $A_n^+$ and $A_n^-$ is the area of triangle $T_n^+$ and $T_n^-$ respectively.

With the above three basis functions, the entries of the L, P, and Z matrices can be defined as:

$$L_{ij} = \frac{\mu}{4\pi} \int_S \int_S \frac{\mathbf{f}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \cdot \mathbf{f}_i(\mathbf{r}')\, ds \quad (3.7)$$

$$P_{ij} = \frac{1}{4\pi\varepsilon} \int_S \int_S \frac{\mathrm{C}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} ds' \mathrm{C}_i(\mathbf{r})\, ds \quad (3.8)$$

$$Z_{ij} = Z_s \int_S \int_S \mathbf{f}_j(\mathbf{r}') \cdot \mathbf{f}_i(\mathbf{r}')\, ds' ds \quad i, j = 1,.., N_e \quad (3.9)$$

where $Z_{ij}$ is non-zero only if edges *i* and edge *j* share a common triangle.

The matrix formulation for the EM part will be:

$$\begin{pmatrix} j\omega\bar{\mathbf{L}}+\bar{\mathbf{Z}} & \bar{\mathbf{A}} & \bar{\mathbf{0}} \\ \bar{\mathbf{0}} & \bar{\mathbf{I}} & \bar{\mathbf{P}} \\ -\bar{\mathbf{A}}^T & \bar{\mathbf{0}} & \bar{\mathbf{D}} \end{pmatrix} \begin{pmatrix} \mathbf{J} \\ \mathbf{V} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \quad (3.10)$$

$\bar{\mathbf{A}}$ is a sparse matrix which describes the adjacency of edges and patches, each row has two non-zero terms which correspond to patches associated with a particular edge. $\bar{\mathbf{D}}$ is a diagonal matrix used to enforce the current and charge continuity equation. The unknowns are the coefficients associated with the current, potential, and charge basis functions.

When coupled with circuits, the formulated EM matrix needs to be extended to include both the circuit part and the EM-circuit connection part, as in equation 3.11.

$$\begin{pmatrix} j\omega\bar{\mathbf{L}}+\bar{\mathbf{Z}} & \bar{\mathbf{A}} & \bar{\mathbf{0}} & \bar{\mathbf{X}} \\ \bar{\mathbf{0}} & \bar{\mathbf{I}} & \bar{\mathbf{P}} & \bar{\mathbf{0}} \\ -\bar{\mathbf{A}}^T & \bar{\mathbf{0}} & \bar{\mathbf{D}} & \bar{\mathbf{0}} \\ \bar{\mathbf{X}}^T & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \overline{\mathbf{MNA}} \end{pmatrix} \begin{pmatrix} \mathbf{J} \\ \mathbf{V} \\ \mathbf{Q} \\ \mathbf{ckt} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{ckt\_ex} \end{pmatrix} \quad (3.11)$$

$\overline{\mathbf{MNA}}$ is the Modified Nodal Analysis matrix of the circuit part, $\bar{\mathbf{X}}$ is a connection matrix which guarantees the current and field continuity at the node where EM structures and circuits are connected. The excitation includes regular voltage and current sources.

## IV Numerical Results

The first example is an interconnect over a ground plane, as in [6]. The geometry is drawn again in Fig.3:

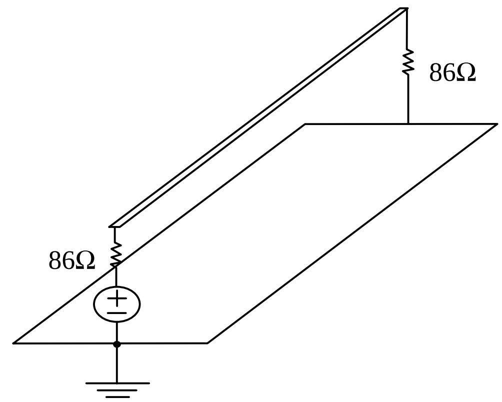


**Figure 3. Interconnect above a solid ground plane**

The interconnect is driven by a voltage source and is terminated by 86 Ohm resistors at both ends. The interconnect is 2.0cm long, 1mm wide and 0.5mm above the ground plane. In this example, the current flow on both the interconnect and ground plane is considered to be two dimensional. The discretized structure and the input impedance are shown in Fig. 4.

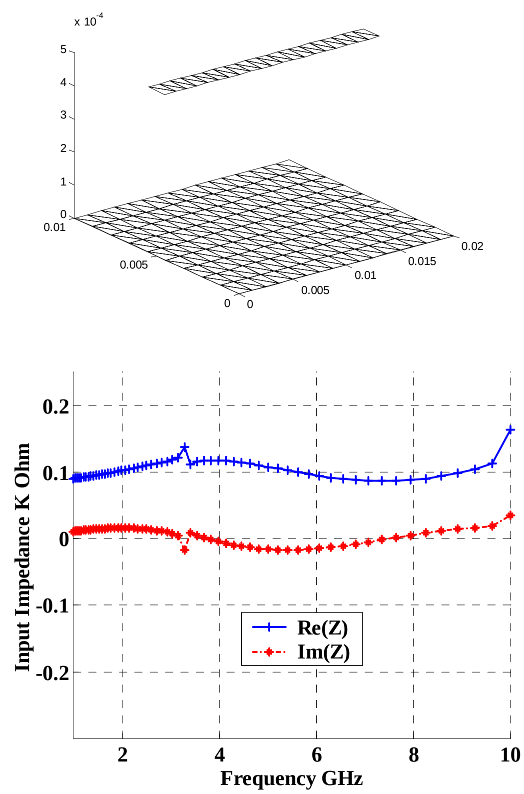


**Figure 4. Triangular meshing of the structure (top)**
**Input impedance of interconnect over a ground plane(bottom)**

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It is interesting to note that the transmission line resonance behavior of this structure is also captured. The finite-sized impedance peaks is due to coarse frequency sampling at resonance.

The second example illustrates that the coupled method can be used for cross talk analysis. Two scenarios are studied here: in one scenario two traces are 0.5mm above the ground plane and 1mm apart, in the other scenario the two traces are 1mm apart at the near end and 2mm apart at the far end. One trace is excited with a 600 ps symmetrical trapezoidal pulse with a 10 ps rise time.

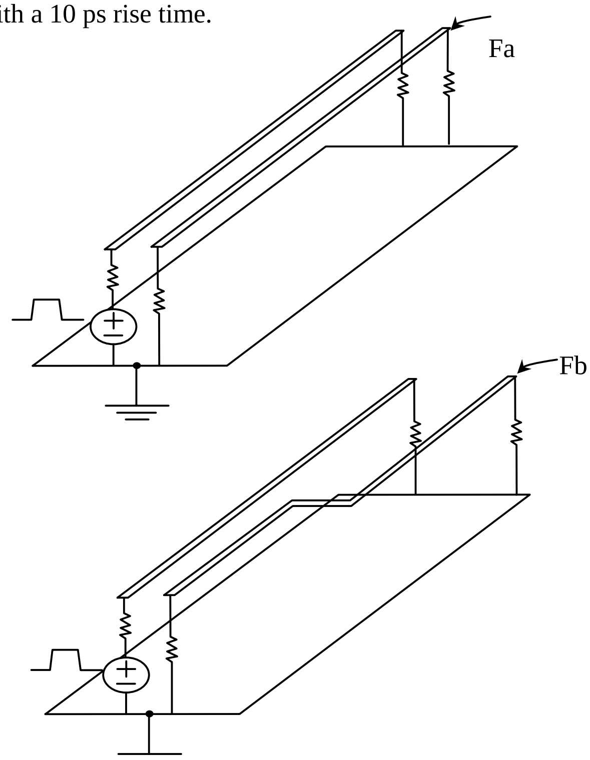


**Figure 5. Two parallel traces(top), two traces with larger distance at far end(bottom)**

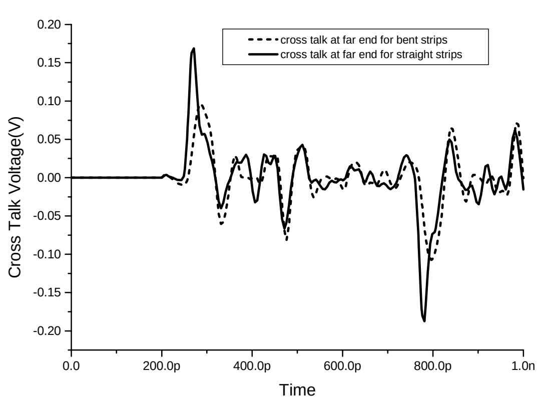


**Figure 6. Cross talk at far end**

As can be seen from Fig. 6, the far end cross talk voltage waveform is weaker for the second case. Also, the time lag between the two crosstalk peaks is the same as the length of the input pulse.

The third example is a spiral inductor of dimensions 200µm×200µm, placed 30µm above the ground plane, as shown in Fig. 7. The observation of interest is the current distribution on the ground plane. The inductor has two turns, and both the line width and the gap width are 20µm.

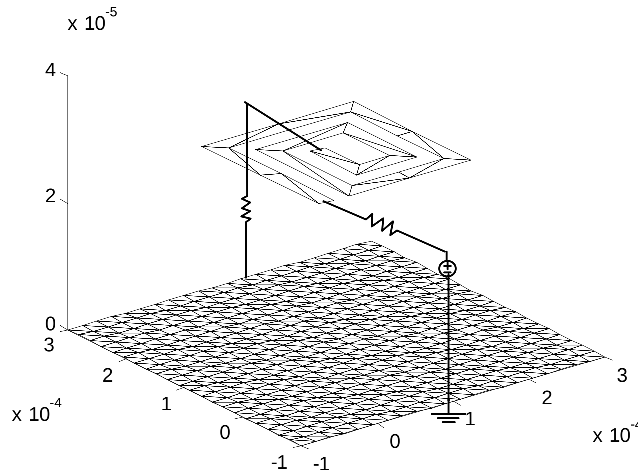

**Figure 7. Spiral inductor above a ground plane**

The coupled EM-circuit simulation gives ground current distribution as in Fig. 8. At 1 GHz the current concentrates below the inductor in order to minimize the inductive impedance of the loop including return current.

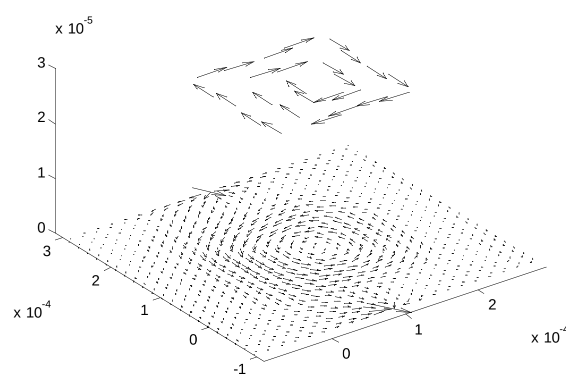

**Figure 8. Current distribution of inductor and ground plane at 1GHz**

## V. Conclusion

In this paper, a generalized PEEC approach based on triangular meshes and well-known RWG basis functions was presented. Also, a SPICE-free coupled EM-circuit formulation was developed in order to solve the dense coupled system outside SPICE. Numerical results were presented to validate the approach and demonstrate its advantages in modeling induced and return current density due to arbitrarily-shaped structures. Furthermore, since the approach is surface-based, it can be used to reduce the numerical computation overhead of circuit-EM analysis by representing very thin structures by a two-dimensional representation.

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