Surface-based PEEC Formulation for Modeling Conductors and Dielectrics in Time and Frequency Domain Combined Circuit Electromagnetic Simulations

Dipanjan Gope†, Albert Ruehli†† and Vikram Jandhyala†

† Department of Electrical Engineering,
University of Washington, Seattle 98195.
{E-mail: *dip, jandhyala@ee.washington.edu*}

†† IBM Research Division,
Yorktown Heights, NY 10598.
{E-mail: *ruehli@us.ibm.com*}

ABSTRACT

The Partial Element Equivalent Circuit (PEEC) formulation is an integral equation based approach for the solution of combined circuit and electromagnetic (EM) problems. In this paper, a surface-based PEEC formulation is presented as an alternative to the existing volume-based method. With the rise in the operating frequencies and the increasing complexity of test structures on boards, packages and chips, a surface based formulation is more efficient for many problems in terms of the number of unknowns generated. The composite conductor dielectric modeling is based on the PMCHWT formulation which is transformed into a PEEC representation using equivalent magnetic and electric circuits connected by mutual coupling. We are interested in both time and frequency domain analyses similar to a Spice type circuit solver.

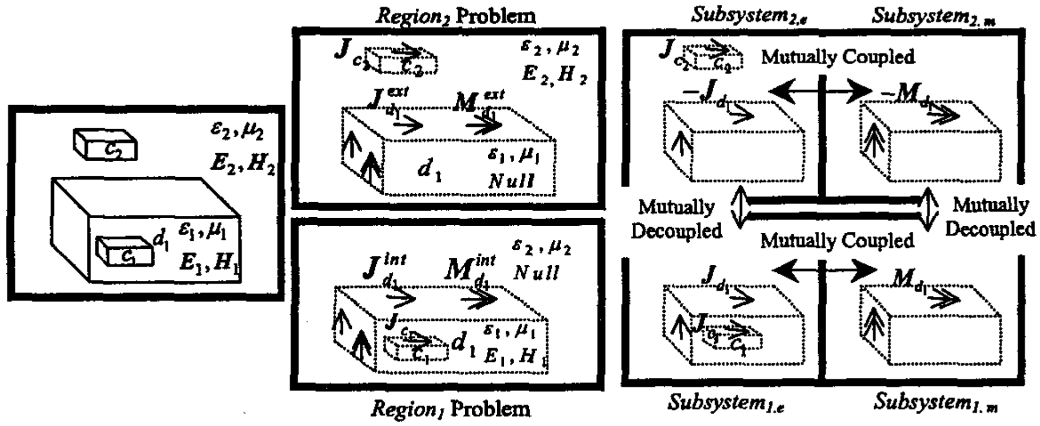
I. Introduction

With the increase in operating frequencies, a combined circuit-EM solution is necessary to accurately predict the electrical performance of a wide range of electronic equipment from mobile products to computer systems. The non-orthogonal PEEC approach [1] has been developed as a volume-based integral equation solver that transforms the EM problems consisting of arbitrarily shaped 3D objects into the circuit domain, by the use of circuit elements such as resistors, partial inductances, capacitances and dependent current and voltage sources, whose values are obtained by solving Maxwell's equations on an appropriately discretized 3D mesh. However, owing to the complexity and the variable nature of the test structures, the modeling scheme for most problems requires extreme flexibility. Volumetric formulations are suitable for certain structures while surface based schemes are more efficient for others. Recently, surface-based PEEC models have been developed for conductors with surface impedance approximations [2-3].

In this work, we develop a surface-based formulation for modeling arbitrarily shaped 3D composite conductor and dielectric structures as a systematic extension to the existing PEEC solvers. The solution of the electromagnetic part is based on the surface equivalence principle [4] and the PMCHWT formulation [5], which has been effectively applied in the past to solve EM scattering problems [6]. The problem has been modeled so as to represent the system by a separate equivalent electric circuit and an equivalent magnetic circuit connected by mutual cross-couplings. The electric circuit represents the electric field equation and the unknowns are the electric node voltages V^e and electric branch currents I^e. The magnetic circuit similarly represents the magnetic field equation and has magnetic node voltages V^m and magnetic branch currents I^m. Both circuits have special lumped equivalent circuits in the form of partial inductances, capacitances and dependent current and voltage sources. The derived partial lumped elements of the two circuits are related by the reciprocity of Maxwell's equations. In a combined circuit-EM solution scheme the derived elements of the equivalent electric circuit are connected to the circuit elements from the input SPICE net-list and the entire system is solved in a single circuit matrix. The solver is therefore capable of handling both circuit excitations, namely voltage and current sources as well as EM excitations in the form of localized or global incident electric fields.

II. Surface Equivalence Principle

The solution process is based on the surface equivalence principle [4], which states that a closed 3D object can be formulated as an interior region problem, and an exterior region problem wherein equivalent magnetic and electric currents on the surface can be effectively used to accurately model the internal and external fields, respectively. Figure 1a shows an example problem that consists of a dielectric object d_1 (ε_1,μ_1) surrounded by a background dielectric (ε_2,μ_2). Two conducting objects, c_1 and c_2, are embedded inside the dielectric object d_1 and the surrounding dielectric, respectively. Using the surface equivalence principle, shown in Fig. 1b the above problem can be represented by 2 equivalent problems: *region₁* problem models the accurate electric and magnetic fields in region 1 (ε_1,μ_1) and the *region₂* problem models the accurate fields in region 2 (ε_2,μ_2).



(a) (b) (c)

Figure 1: (a) Physical Problem (b) Surface Equivalent Problems (c) PEEC Subsystem Representations

The electric and magnetic current densities are related by:

$$J_{d_1}^{int} = -J_{d_1}^{ext} \; ; \; M_{d_1}^{int} = -M_{d_1}^{ext} \tag{2.1}$$

The surface equivalent problems can thus be represented in a subsystem format as demonstrated in Fig. 1c, which enables a clear visualization of the mutual couplings between the equivalent currents. $Region_i$ is decomposed into $subsystem_i$ and $subsystem_{i,m}$, which represent the electric and magnetic currents, respectively. Every dielectric-dielectric interface S is therefore associated with 2 regions S_{int} and S_{ext}. Consequently, from a subsystem perspective, S is associated with 2 electrical subsystems, namely $subsystem_{Sint,e}$ and $subsystem_{Sext,e}$, and 2 magnetic subsystems namely $subsystem_{Sint,m}$ and $subsystem_{Sext,m}$. The insides of conductors are not modeled as separate regions and the surfaces of conductors only support electrical currents. Unlike the volumetric formulation where all EM currents are mutually coupled, in the surface case electrical or magnetic quantities like current and charge are mutually decoupled across subsystems, i.e. $subsystem_{i,e}$ and $subsystem_{i,m}$ are mutually coupled but are individually decoupled from $subsystem_{j,e}$ and $subsystem_{j,m}$, where $i \neq j$.

III. Integral Equation

The PEEC formulation uses an integral equation formulation based on PMCHWT [5]. The boundary condition applied on a given surface S is given by:

$$\hat{t} \cdot \left[E^i + E_2\right] = \hat{t} \cdot E_1; \;\; \hat{t} \cdot H^i + \hat{t} \cdot H_2 = \hat{t} \cdot H_1 \tag{3.1}$$

where $\hat{t}$ is a non-unique unit vector tangential to the surface, E_1 and H_1 are the scattered electric and magnetic fields computed for $Subsystem_{Sint,e}$ and $Subsystem_{Sint,m}$, E_2 and H_2 are the scattered electric and magnetic fields computed for $Subsystem_{Sext,e}$ and $Subsystem_{Sext,m}$, and E^i and H^i are the incident electric and magnetic fields in the exterior region. If J_1 and M_1 represent the electric and magnetic current densities and q_1^e and q_1^m represent the electric and magnetic charge in $Subsystem_{Sint}$ and $Subsystem_{Sint,m}$, respectively, then using (2.1) the scattered fields at any point r_g in the global coordinate system are given by:

$$E_{1,2}(r_g) = -j\omega A_{1,2}(r_g) - \nabla\phi_{1,2}^e(r_g) - \frac{1}{\varepsilon_{1,2}}\nabla\times F_{1,2}(r_g)\, ; \quad H_{1,2}(r_g) = -j\omega F_{1,2}(r_g) - \nabla\phi_{1,2}^m(r_g) + \frac{1}{\varepsilon_{1,2}}\nabla\times A_{1,2}(r_g) \tag{3.2}$$

where:

$$A_{1,2}(r_g) = \mu_{1,2}\int_S G_{1,2}(r_g, r_{g'})[+,-]J_1(r_{g'})dr_{g'}\, ; \qquad F_{1,2}(r_g) = \varepsilon_{1,2}\int_S G_{1,2}(r_g, r_{g'})[+,-]M_1(r_{g'})dr_{g'} \tag{3.3}$$

$$\phi_{1,2}^e(r_g) = \frac{1}{\varepsilon_{1,2}}\int_S G_{1,2}(r_g, r_{g'})[+,-]q_1^e dr_{g'}\, ; \qquad \phi_{1,2}^m(r_g) = \frac{1}{\mu_{1,2}}\int_S G_{1,2}(r_g, r_{g'})[+,-]q_1^m dr_{g'} \tag{3.4}$$

The free-space full-wave Green's function is given by

$$G_{1,2}(r_g, r_{g'}) = \frac{1}{4\pi}\frac{\exp(-j\omega\sqrt{\mu_{1,2}\varepsilon_{1,2}}\,|r_g - r_{g'}|)}{|r_g - r_{g'}|} \tag{3.5}$$

Using (3.2) in boundary condition (3.1), the basic integral equations are formulated as:

$$\hat{t}\cdot \boldsymbol{E}^i(r_g) = \hat{t}\cdot\left(j\omega\left[A_1(r_g)+A_2(r_g)\right] + \nabla\left[\phi_1^e(r_g)+\phi_2^e(r_g)\right] + \left[\frac{1}{\varepsilon_1}\nabla\times F_1(r_g) + \frac{1}{\varepsilon_2}\nabla\times F_2(r_g)\right]\right) \tag{3.6}$$

$$\hat{t}\cdot \boldsymbol{H}^i(r_g) = \hat{t}\cdot\left(j\omega\left[F_1(r_g)+F_2(r_g)\right] + \nabla\left[\phi_1^m(r_g)+\phi_2^m(r_g)\right] - \left[\frac{1}{\mu_1}\nabla\times A_1(r_g) + \frac{1}{\mu_2}\nabla\times A_2(r_g)\right]\right) \tag{3.7}$$

IV. *Circuit Representation*

In section III, the integral equations (3.6) and (3.7) have been formulated. In this section, we determine the discrete equivalent circuits for non-orthogonal geometries. The contribution of each element to the circuit matrix is captured by what is called the (S)-PEEC matrix stamp. The basic PEEC discretization is explained in [1] and is briefly illustrated in Fig. 2. Evidently both inductive and capacitive cells are quadrilaterals for the surface based formulation.

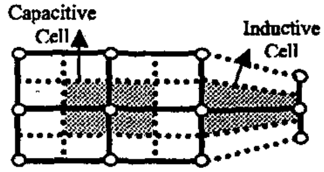


Figure 2: PEEC Discretization

Equation (3.6) is modeled by an electric circuit with electric current I^e and electric voltage V^e as unknowns, where:

$$J_a = \frac{h_a}{\left|\frac{\partial r_g}{\partial a}\times\frac{\partial r_g}{\partial b}\right|} I_a^e; \quad V_a^e = \int_a\int_b \frac{h_a}{\left|\frac{\partial r_g}{\partial a}\times\frac{\partial r_g}{\partial b}\right|}\hat{a}\cdot E\left|\frac{\partial r_g}{\partial a}\times\frac{\partial r_g}{\partial b}\right| da\,db \tag{4.1}$$

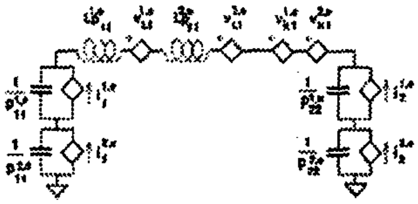


Figure 3: (S)PEEC unit electric-cell

using the same notations as in [1]: *a, b, c* represent the local coordinate system and $h_a = \left|\frac{\partial r_g}{\partial a}\right|$ represents the magnitude of the tangential vector. A detailed description of the equivalent electric circuit for a unit (S)PEEC dielectric cell is demonstrated in Fig. 3. The superscript in the notation for the circuit elements indicates the subsystem from which the element originated. The subscript denotes the cell numbers associated with the element. The first of the 3 pairs of terms in (3.6) are represented by the self-inductance L_p and the mutual inductances that are grouped together in the current controlled voltage source v_L. Similarly, the second pair is represented by the self-capacitance $1/p$ and the mutual effects captured by the current controlled current sources *i*. The partial inductive and potential elements are given by:

$$Lp_{aa'}^{SS} = \mu_{SS}\int_a\int_b\int_{a'}\int_{b'}(\hat{a}\cdot\hat{a}')h'_a G_{SS}(r_g, r'_g)\,da'db'h_a\,da\,db\,; \qquad Pn_{aa'}^{SS} = \frac{1}{\varepsilon_{SS}}\int_a\int_b\int_{a'}\int_{b'} G_{SS}(r_g, r'_g)\,da'db'da\,db \tag{4.2}$$

where *SS* represents the subsystem under consideration. The last pair of terms in (3.6) represents the effect on the electric circuit from its magnetic counterpart and is captured by current controlled voltage source v_k.

$$v_{k_{aa'}}^{SS} = \int_a\int_b h_a\hat{a}\cdot\int_{a'}\int_{b'} h'_a\left[\hat{a}'\times\nabla' G_{SS}(r_g, r'_g)\right]da'db'da\,db \tag{4.3}$$

$$= \frac{1}{4\pi}\int_a\int_b h_a\hat{a}\cdot\int_{a'}\int_{b'} h'_a\left[\hat{a}'\times\frac{\exp(-j\omega\sqrt{\mu_{SS}\varepsilon_{SS}}\,|r_g - r_{g'}|)}{|r_g - r_{g'}|^3}(r_g - r_{g'})\right]da'db'da\,db + \frac{j\omega}{4\pi}\int_a\int_b h_a\hat{a}\cdot\int_{a'}\int_{b'} h'_a\left[\hat{a}'\times\frac{\exp(-j\omega\sqrt{\mu_{SS}\varepsilon_{SS}}\,|r_g - r_{g'}|)}{|r_g - r_{g'}|^2}(r_g - r_{g'})\right]da'db'da\,db$$

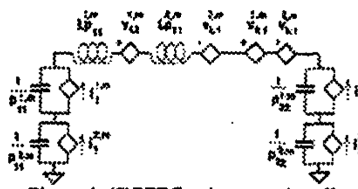


Figure 4: (S)PEEC unit magnetic-cell

Unlike v_L, for v_k the controlling currents are magnetic as shown in Fig. 4, and the dependence has both a frequency independent term as well as a linear frequency term.

Equation (3.7) is modeled similarly using a magnetic circuit. From the standpoint of the reciprocity of Maxwell's equations, the circuit for Fig. 4 can be obtained from the electrical counterpart formulated in reciprocal media, i.e., the dielectric constant (ε) replaced by the permeance (μ) and vice-versa.

V. Numerical Experiments

In the first example, we model a transmission line structure as shown in Fig. 5a. Each of the pair of traces is $100\,\mu m$ in length and has a square cross-section of $25\,\mu m \times 25\,\mu m$. An ac current source of 1A is connected between the traces on one side while the other side is electrically shorted. The surface impedance approximation is used for the surface formulation and therefore magnetic circuits are not required in the modeling. The current densities on the structure for 1GHz. are plotted in Fig. 5b, and it can be observed that the current flows in a smaller loop to reduce the inductance. The inductance and the resistance of the structure obtained from the surface and volume formulations are compared in Fig. 5c and 5d. The surface impedance model can be seen to be accurate for frequencies where the skin depth is smaller than the conductor cross sections. At those frequencies the volume formulation requires very fine discretization towards the edges. Therefore a considerable speedup is obtained with the surface model at high frequencies, e.g., in this case a 50x speedup is obtained at 5GHz.

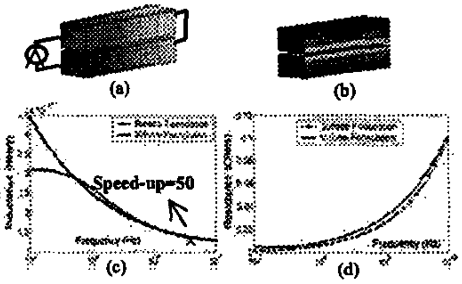


Figure 5: (a) Transmission line structure and excitation (b) Current density at 1GHz. (c) Inductance vs. frequency (d) Resistance vs. frequency

In the next example, we validate the surface formulation against volumetric results for the case of a dielectric embedded between two conductors. All structures have $1mm \times 1mm$ cross-sections. The conductors are $0.1mm$ thick each and the dielectric is centrally placed between them as shown in Fig. 6a. For experimentation the dielectric thickness used are $0.05mm$ and $0.09mm$. The imaginary impedance of this capacitive setup is plotted in Fig. 6b for both thick and thin dielectrics using the surface and the volume models.

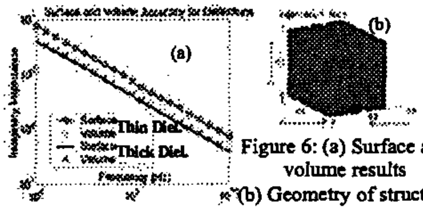


Figure 6: (a) Surface and volume results (b) Geometry of structure

In the last example the surface model is validated for touching structures as in the transmission line setup of Fig. 7a. The cross-section of the conductors is $0.001mm \times 0.02mm$ and that of the sandwiched dielectric ($\varepsilon_r = 20$) is $0.018mm \times 0.02mm$. All structures are $0.3mm$ long. In Fig. 7c, the frequency response of the structure is shown for the surface and the volume models. The plot demonstrates reasonable agreement between the results.

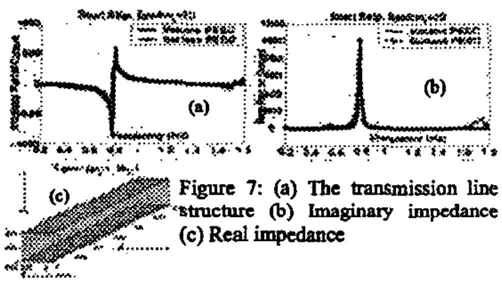


Figure 7: (a) The transmission line structure (b) Imaginary impedance (c) Real impedance

VI. Conclusions

In this paper, the surface based (S)PEEC formulation for conductors and dielectrics is presented. The surface equivalence principle is used to derive the electric and magnetic circuits required. Though the number of unknowns appears to double due to the use of both electric and magnetic circuit, the small number of basis functions owing to a surface only formulation leads to time and memory savings for thick structures. Furthermore, like the original (V)PEEC method this can support both time and frequency domain analysis.

References:

[1] A.E.Ruehli, G.Antonini, J.Esch, J.Ekman, A.Mayo and A.Orlandi, "Nonorthogonal PEEC formulation for time- and frequency-domain EM and circuit modeling", *IEEE Trans. on Electromagnetic Compatibility*, Volume: 45, Issue 2, pp. 167-176, May 2003.

[2] A.Rong, A.C.Cangellaris and L.Dong, "Comprehensive broadband electromagnetic modeling of on-chip interconnects with a surface discretization-based generalized PEEC model", *Proc. IEEE Elec. Perf. of Electronic Packaging conf.*, pp. 367-370, Princeton, NJ, Oct. 2003.

[3] Yong Wang, Dipanjan Gope, Vikram Jandhyala, and C.J. Richard Shi, "Integral Equation-Based Coupled Electromagnetic-Circuit Simulation in the Frequency Domain," *Proc. IEEE APS-URSI Ohio*, vol. 3, pp. 328-331, June 2003.

[4] A.F.Peterson, S.L.Ray and R.Mittra, Computational Methods for Electromagnetics, IEEE/Oxford University Press Series on Electromagnetic Wave Theory, 1998.

[5] A.J. Poggio, E.K. Miller, Y. Chang, R.F. Harrington, T.K Wu, L.L. Tsai. PMCHWT combined integral equation formulation named after the contributors, 1973-1977. Original collective work.

[6] K.Umashankar, A.Taflove and S.Rao, "Electromagnetic scattering by arbitrary shaped three-dimensional homogeneous lossy dielectric objects", *IEEE Trans. on Antennas and Propagation*, Volume: 34, Issue 6, pp. 758-766, Jun 1986.