

# Combined Multilevel FMM – QR Algorithm For Broadband Applications

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## Abstract

This paper presents a combined multilevel FMM-QR approach applicable to three-dimensional scattering problems of widely varying electrical sizes. The classic MLFMA suffers breakdown at low frequencies [1], while QR based methods become inefficient at high frequencies [2]. The presented algorithm combines these two methods to achieve stability at all frequencies, and at the same time preserves the  $O(N\log N)$  complexity of setup, memory and matrix-vector products. Examples demonstrating time and memory requirements are presented, and the efficient nature of the overall method at all frequencies is also demonstrated.

## 1. Introduction

Fast iterative methods [3] are increasingly becoming popular for solving 3-D scattering problems with large number of unknowns. These methods use oct-tree based geometric decomposition and exploit the low rank nature of far-field interactions to compress dense Method of Moments (MoM) [4] systems to achieve linear memory and time scaling with increasing discretization. The multilevel fast multipole algorithm (MLFMA) [3] is commonly used for solving electrically large scattering problems, while QR based compression schemes [2] have been used to solve electrically small problems. The MLFMA suffers breakdown when size of the interacting cubes becomes small compared to the wavelength, which makes it inapplicable in the low frequency range although researchers have used modified methods to attempt to overcome this disadvantage [1,3]. On the other hand, QR based methods fail to achieve compression for oscillatory kernels thus making them inapplicable at high frequencies. It is reasonable to guess that a combination of these two methods will make the problem stable at all frequencies. This paper does that by combining their best features into a single algorithm, uses the same oct-tree structure and preserves the  $O(N\log N)$  complexity, which is a common feature of both the methods.

## 2. The FMM-QR Algorithm

This section describes the FMM-QR combined algorithm and analyzes its complexity.

1) *Oct-tree Decomposition*: The 3-D computational domain is broken down into a hierarchical *oct-tree* structure. A 0<sup>th</sup> level starting cube is chosen enclosing the given geometry and is split into eight child cubes, which form the 1<sup>st</sup> level. This splitting process is repeated recursively until  $L$  levels are generated, depending on the problem

size. Let  $(ka)^l$  denote the electrical size of a cube at level  $l$ . For each cube, neighbor list and interaction list are maintained [2,3].

2) *Decide kind of level*: This is the key step of this algorithm. For each level  $l$ , decide whether this is a ‘FMM’ or a ‘QR’ level. If  $(ka)^l > cutoff$  then  $l$  is a FMM level, otherwise it is a QR level. The idea is, if level  $l$  is a QR level then the contributions of the interaction list can be compressed using the QR scheme described in [2]. Otherwise, the cubes at level  $l$  interact via FMM operators. Generally, the cutoff size is chosen to be 0.2. Let  $j_{FMM}$  denote the finest FMM level.

3) *Setup operators*:

For level  $i = j_{FMM}$  to 2

Setup multilevel FMM operators as described in [3], i.e.,

i) If  $i = j_{FMM}$  form the Q2M and L2P operators.

ii) For all FMM levels compute the translation and interpolation operators.

End

For level  $i = L$  to  $j_{FMM} + 1$

Form merged interaction list and perform QR compression [2].

End

Finally, at level  $L$  compute the near-field contributions directly.

Depending on the electrical size of the problem there can be three cases – no FMM levels (all QR levels), no QR levels (all FMM levels), and both FMM and QR levels. Thus, all operators are free of breakdown and at the same time compression of the interactions is achieved. Since the number of operations is bounded by  $O(N \log N)$  in both algorithms [2, 3], the net setup cost is also  $O(N \log N)$ .

4) *Matrix-vector product*:

For level  $i = j_{FMM}$  to 2

Perform multilevel FMM matrix vector products [3]

End

For level  $i = L$  to  $j_{FMM} + 1$

Perform matrix-vector products using QR compressed interaction matrices.

End

At level  $L$  compute the near-field matrix-vector products.

Again, all of the involved steps take  $O(N \log N)$  operations, preserving linear nature of the matrix-vector product. Fig. 1 explains the idea of the multilevel algorithm diagrammatically. Notice, that in the FMM levels there is a tree ascent and a tree descent step during the matrix-vector product. While in the QR levels, there is no tree traversal during the matrix-vector product since each interaction is compressed separately and there is no interaction between levels [2].

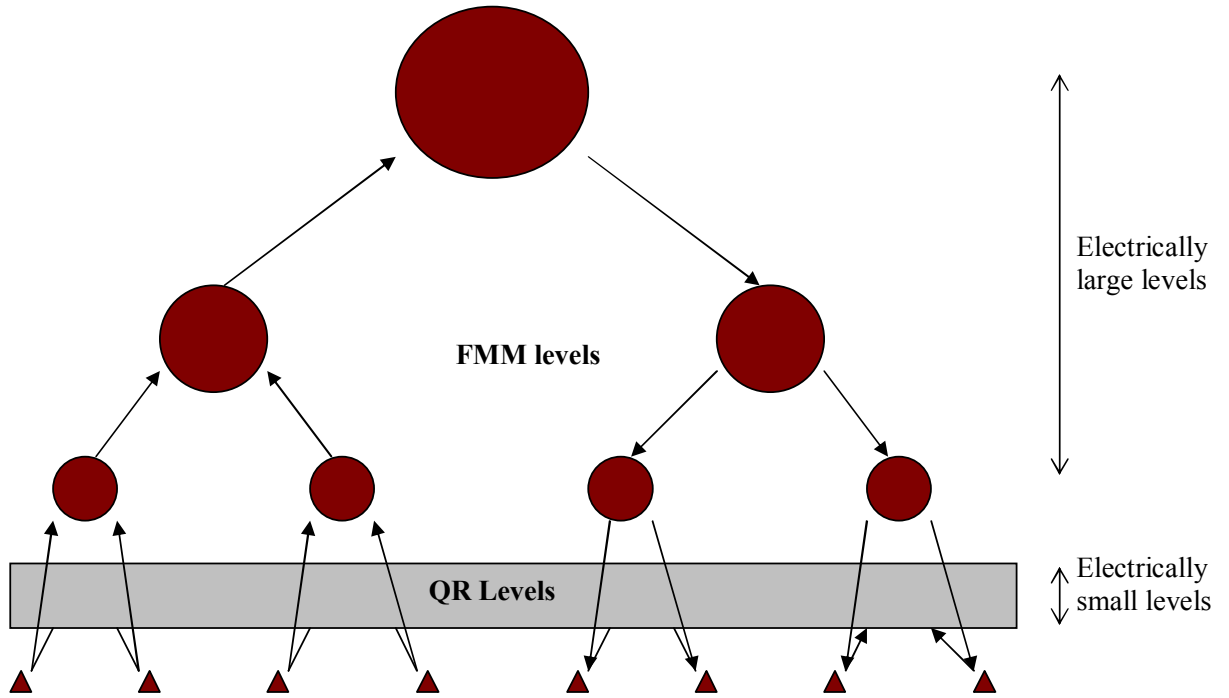


Fig 1: Schematic representation of the FMM-QR algorithm

Table 1: Memory and time scaling for a sphere of radius 1mm (at 22 GHz)

#patches(N)	Time(sec)	Memory(B)	#Levels	#FMM levels	# QR levels
5000	1.7	174M	3	3	0
6500	3.16	320M	4	3	1
11000	6.29	674M	4	3	1
21000	12.05	1.16G	5	3	2
30000	18.5	1.7G	5	3	2

Table 2: Frequency dependence of FMM-QR levels for a 1mm X 1mm plate

N	Frequency	#levels	Rel. error with LU	#FMM levels	#QR levels
3120	80GHz	3	3.34e-2	2	1
3120	100GHz	3	4.445e-2	3	0

### 3. Numerical Results

The presented algorithm was tested on a 3.2GHz (2GB RAM) Linux platform. A sphere of 1mm radius was excited by a plane wave of frequency 22 GHz. Table 1 documents

the memory and single matrix-vector product time requirements of the tested code. Note that as the number of levels was increased, the finer levels were QR levels and the higher levels were FMM levels. Table 2 shows two examples of the relative errors of the current distribution when the EFIE was solved using FMM-QR for a  $1\text{mm} \times 1\text{mm}$  plate, with different FMM and QR levels. This demonstrates the stability of the algorithm at varying frequency range.

#### 4. References

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