Time Domain Analysis using Higher-Order Wires

Nathan J. Champagne[1*], Vikram Jandhyala[2], Benjamin J. Fasenfest[2],
John D. Rockway[3], and Donald R. Wilton[4]
[1]Louisiana Tech University, Ruston, LA 71272
[2]University of Washington, Seattle, WA 98195
[3]Lawrence Livermore National Laboratory, Livermore, CA 94550[†]
[4]University of Houston, Houston, TX 77204-4005
champagn@latech.edu

Introduction

The electromagnetic code EIGER (Electromagnetic Interactions GEneRalized) [1] has been modified to include temporal analysis methods. The time domain electric field integral equation (TD-EFIE) is used to model scattering and radiation from perfect electric conductors. The most popular solution scheme found in the literature is the implicit marching-on-in-time (MOT) algorithm, which suffers none of the traditional late-time instabilities normally associated with the TD-EFIE. As several researchers have discussed, an implicit MOT scheme, specialized temporal basis functions, and accurate time-delayed potential integrations will provide numerically accurate, stable results [2–4]. In this paper, we consider each of these three aspects specifically for the time domain analysis of wires.

The key components of the EIGER numerical analysis tool, and their roles, are: elements – to describe the geometry, basis functions – to interpolate the unknown currents locally, and singular quadratures and weights – to accurately integrate the underlying physics. Wire elements incorporating curvature, for example, provide ample resolution of geometry. Finally, the potential integrals incorporating time retardation involve the static 1/R singularity or near singularity, and therefore require special numerical considerations for their evaluation. For wires, a simple and efficient numerical procedure using a singularity cancellation method discussed in [5, 6] is used.

TD-EFIE

To analyze pulsed scattering or broadband radiation from wires, the unknown currents are determined via the solution of the time domain electric field integral equation (TD-EFIE) given by

$$[\mathbf{E}^i(\mathbf{r},t)+\mathbf{E}^s(\mathbf{r},t)]_{\tan}=0, \quad \mathbf{r}\in S. \tag{1}$$

The excitation $\mathbf{E}^i(\mathbf{r},t)$ may be either an incident transient electric field or time-dependent voltage source. The scattered electric field is expressed in terms of the magnetic vector and the electric scalar potentials as

[†] This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under contract No. W.7405-ENG-48.

0-7803-8883-6/05/$20.00 ©2005 IEEE

$$\mathbf{E}^s(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t) - \nabla\phi(\mathbf{r},t), \tag{2}$$

where the potentials are represented as

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu}{4\pi}\int_S \frac{\mathbf{J}(\tau,\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dS' \tag{3}$$

and

$$\phi(\mathbf{r},t) = -\frac{1}{4\pi\varepsilon}\int_S \int_0^\tau \frac{\nabla\cdot\mathbf{J}(t',\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dt'\,dS' \tag{4}$$

and $\tau = t - |\mathbf{r}-\mathbf{r}'|/c$ is the time delay. The electric current on the wire is approximated as

$$\mathbf{J}(\mathbf{r},t) = \sum_{n=1}^{N}\sum_{j=1}^{T} J_{n,j}\,\mathbf{I}_n(\mathbf{r})T_j(t), \tag{5}$$

where $J_{n,j}$ is the unknown current coefficient at time t_j, $I_n(\mathbf{r})$ is the wire basis function, and $T_j(t)$ is the temporal basis function. Substituting (5) into (2) and delta testing in time yields the following system matrix:

$$\sum_{j=1}^{T} Z_{i-j} I_j = V_i\,. \tag{6}$$

An implicit time-stepping scheme ($\Delta t > \alpha R_{\min}/c$) is then used to determine the unknown current coefficients for all time where α is a parameter ensuring an implicit time step.

Numerical Implementation

The singular integrals that arise in the time-delayed vector and scalar potentials due to source distributions that are represented by basis functions require special numerical considerations for their evaluation. For thin wires, a simple and efficient numerical procedure using a singularity cancellation method was implemented in [5, 6]. In this approach, a change of variables is employed such that the Jacobian of the transformation cancels the singularity. The resulting integrand involves a kernel singularity of the form $\ln(R)$ that is handled using the scheme proposed in [7].

A scheme to employ higher-order elements and basis functions to accurately resolve the geometry and current has been implemented similar to the approach taken for basis functions in [8]. These elements may be of arbitrary order and will more accurately resolve the local variations of the geometry. Wire segments need at least three points to define a curved segment.

Results

For radiation and scattering problems, some popular right hand sides involve delta-gap sources and plane wave excitations given in functional form as

$$\mathbf{E}^i(\mathbf{r},t) = \begin{cases} \mathbf{E}_0 \dfrac{4}{\sqrt{\pi}T} e^{\left[-\frac{4}{T}(ct-ct_0-\mathbf{r}\cdot\mathbf{k})\right]^2} \\ V_0 \dfrac{4}{\sqrt{\pi}T} e^{\left[-\frac{4}{T}(ct-ct_0)\right]^2} \end{cases} . \tag{7}$$

A current for a center-fed dipole antenna (T = 2 lm, ct_0 = 3 lm) modeled with 10 straight segments is shown in Figure 1. The plots are for two cases: $\alpha = 1$ and $\alpha = 2$. The results show good agreement with the data in [2] and demonstrate the convergence behavior for the two choices of α. A wire loop with a one meter diameter is located in the xy plane and is centered about the origin. It is illuminated by a plane wave ($\phi = 0°$, $\theta = 90°$, $E_\phi = 1$ V/m, T = 4 lm, ct_0 = 6 lm) [9]. The currents for a loop modeled with linear elements (q = 1) and quadratic segments (q = 2) are shown in Figure 2. Note that since linear basis functions were used, the number of quadratic segments needed for convergence was similar to that needed for straight wires (30 elements).

Summary

A stable time-domain analysis of wires using higher-order analysis for wires has been implemented in EIGER. Advanced integration techniques for evaluating the wire kernels accurately integrate the time-delayed potentials found in the TD-EFIE. Using higher-order elements along with higher-order basis functions may reduce the number of elements needed for modeling the curvature of a wire structure.

References

[1] R.M. Sharpe, J.B. Grant, N.J. Champagne, W.A. Johnson, R.E. Jorgenson, D.R. Wilton, W.J. Brown, J.W. Rockway, "EIGER: Electromagnetic Interactions GEneRalized", *1997 IEEE AP-S International Symposium and North American URSI Radio Science Meeting*, Montreal, Canada, pp. 2366–2369, July 1997.

[2] B. H. Jung, T. K. Sarkar, Z. Ji, and Y. Chung, "Time-domain analysis of conducting wire antennas and scatterers," *Microwave & Optical Technology Letters*, vol. 38, no. 6, pp. 433–436, Sept. 2003.

[3] D. S. Weile, G. Pisharody, N. Chen, B. Shanker, and E. Michielssen, "A novel xcheme for the solution of the time-domain integral equations of electromagnetics," *IEEE Trans. Ant. and Prop.*, vol. 52, no. 1, pp. 283–295, Jan. 2004.

[4] C.Yang, G. Ouyang, and V. Jandhyala, "Integral equation based time domain coupled EM-circuit simulation for packaged conductors and dielectrics.," *Electrical Performance of Electronic Packaging*, 27–29, pp. 371–374, Oct. 2003.

[5] M. A. Khayat and D. R. Wilton, "Evaluation of singular and near-singular potentials," *2003 IEEE AP-S International Symposium and North American URSI Radio Science Meeting*, Columbus, OH, June 2003.

[6] M. Khayat and D. R. Wilton, "Revisiting the evaluation of potential integrals," *2003 International Conference on Electromagnetics in Advanced Applications*, Torino, Italy, pp. 83–86, Sept. 2003.

[7] J. Ma, V. Rokhlin, and S. Wandzura, "Generalized Gaussian Quadrature rules for systems of arbitrary functions," *Siam J. Numer. Anal.*, 33, pp. 971–996, 1996.

[8] R. D. Graglia, D. R. Wilton and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," *IEEE Trans. Antennas and Propagation*, vol. 45, no. 3, pp. 329–342, Mar. 1997.

[9] Z. Ji, T. K. Sarkar, B. H. Jung, Y.-S. Chung, M. Salazar-Palma, and M. Yuan, "A stable solution of time domain electric field integral equation for thin-wire antennas using Laguerre polynomials," *IEEE Trans. Antennas and Propagation*, vol. 52, no. 10, pp. 2641–2649, Oct. 2004.

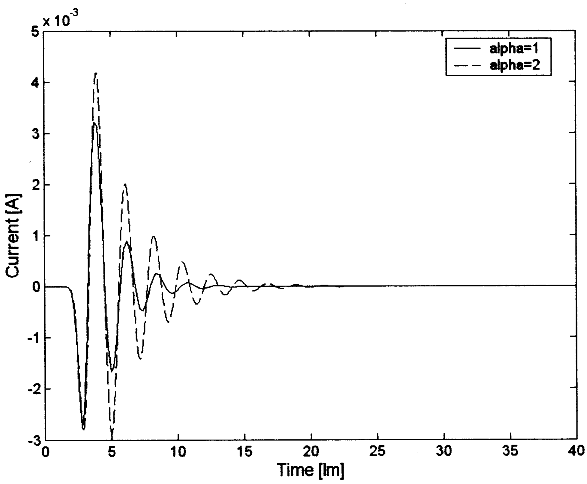

Figure 1. Plot of current at center of a 1 meter dipole.

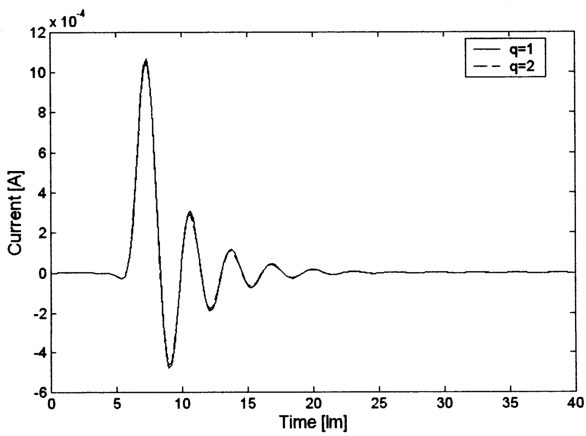

Figure 2. Current at ($x = 0.5, y = 0.0$) on a 1m diameter loop centered about the origin in the xy plane.