Surface-Based Broadband Electromagnetic-Circuit Simulation of Lossy Conducting Structures in Microelectronic Circuits

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Introduction

Rapid growth in the switching speed, and packing density of integrated digital, analog, RF, and microwave circuits has increased the importance of accurate electromagnetic (EM) modeling. While modeling an entire system with full-wave EM simulation is impractical due to the computational complexity, it is important to model the distributed effects in sensitive portions and couple them to the lumped-element models from circuit simulation. This coupled EM-circuit approach is particularly important for field-aware design, such as hot spot prediction, decoupling capacitor placement, and ground bounce, where port models will not suffice.

The volume integral equation (IE) based partial element equivalent circuit (PEEC) approach [1], works well for interconnects at frequencies where volume conduction is the dominant current flow, but requires an explicit modeling of skin depth in the form of a frequency dependent volumetric discretization at higher frequencies. Surface IE based methods are typically well suited for interconnect analysis due to their frequency independent discretization for modeling the skin effect. Such methods, using scalar Green's functions have been implemented for RLC extraction [2]. Fully-coupled circuit-EM solution [3] has been reported for perfect electric conductors and also for lossy conducting material at high frequency where the surface impedance assumption [4] is valid. The surface impedance approach, though very useful for high frequency applications, fails to capture the low frequency effects due to volumetric current flow through the conductor and hence is unsuitable for broadband digital applications. The method presented here is focused towards a frequency domain broadband simulation of arbitrarily shaped lossy objects coupled to circuits using surface integral equations and a unified coupling matrix approach, and presents itself as a smooth and seamless transition from high-frequency surface impedance approximations to low-frequency volumetric methods, in a single formulation.

Formulation

A two-region PMCHWT [5] formulation, formed by decomposition of the problem into an equivalent exterior and interior problem using the background and the conducting medium Green's functions respectively, is used. For an interconnect with finite conductivity, the interior electric field $\mathbf{E}$ does not completely vanish, hence equivalent magnetic current sources $\mathbf{M}$ exist on the boundary, along with the equivalent electric current source $\mathbf{J}$ as,

$$\mathbf{M} = \hat{\mathbf{n}} \times \mathbf{E} \neq 0 \tag{1}$$

where $\hat{\mathbf{n}}$ is the unit normal to the conductor surface. The boundary condition in the tangential components of the electric and magnetic field on the surface of the EM object can be expressed in terms of the potentials as,

$$\begin{aligned}&\left(-j\omega\mathbf{A}_e - \nabla\phi_e - \frac{1}{\varepsilon_e}\nabla\times\mathbf{F}_e + \mathbf{E}_e^{exc}\right)_{\tan}\\ &= \left(-j\omega\mathbf{A}_i - \nabla\phi_i - \frac{1}{\varepsilon_i}\nabla\times\mathbf{F}_i + \mathbf{E}_i^{exc}\right)_{\tan}\end{aligned} \tag{2a}$$

$$\begin{aligned}&\left(-j\omega\mathbf{F}_e - \nabla\psi_e + \frac{1}{\mu_e}\nabla\times\mathbf{A}_e + \mathbf{H}_e^{exc}\right)_{\tan}\\ &= \left(-j\omega\mathbf{F}_i - \nabla\psi_i + \frac{1}{\mu_i}\nabla\times\mathbf{A}_i + \mathbf{H}_i^{exc}\right)_{\tan}\end{aligned} \tag{2b}$$

where the suffix e, i stand for the exterior and interior medium respectively, **A**, **F** denote the Magnetic and electric vector potential and ϕ , ψ are the electric and magnetic scalar potential. $\varepsilon_e, \varepsilon_i$ are the permittivities and μ_e , μ_i are the permeabilities of he exterior and the interior media respectively. ω is the angular frequency. E_m^{exc} and H_m^{exc} are the incident electric and magnetic field in the m^{th} (e or i) medium. The potentials are related to the electric current density $\mathbf{J}_m$, magnetic current density $\mathbf{M}_m$,electric charge density ρ_m, and magnetic charge density ζ_m in the m^{th} medium.

$$\begin{pmatrix} \mathbf{A}_m \\ \phi_m \\ \mathbf{F}_m \\ \psi_m \end{pmatrix} = \begin{pmatrix} \mu_m \\ \dfrac{1}{\varepsilon_m} \\ \varepsilon_m \\ \dfrac{1}{\mu_m} \end{pmatrix} \cdot * \, G_m \otimes \begin{pmatrix} \mathbf{J}_m \\ \rho_m \\ M_m \\ \zeta_m \end{pmatrix} \tag{3}$$

$\otimes$ and $\cdot *$ are the convolution and the element by element product operations respectively. G_m is the Green's function in the m^{th} medium, and is given by

$$G_m(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_m|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{4}$$

k_m is the wave-number in the m^{th} medium. It is important to note that for highly lossy media, k has a strong negative imaginary part, hence the Green's function show a rapid spatial decay and polar-coordinate based quadrature is required to carry out the Green's function convolution [6]. The current densities are related to the corresponding charge densities by the continuity equation as

$$\begin{pmatrix} \nabla \cdot J_m \\ \nabla \cdot K_m \end{pmatrix} = -j\omega \begin{pmatrix} \rho_m \\ \zeta_m \end{pmatrix} \tag{5}$$

The coupling between the EM object and the circuit is achieved through a set of coupling currents (Fig. 1), from the circuit nodes to the EM object, where the circuit node is connected to the corresponding faces of the EM object. The coupling currents are related to the circuit currents by KCL at the connection nodes, and with the equivalent electric current density of the connection patches by a modified continuity equation

$$\nabla_S \cdot \mathbf{J}_m(\mathbf{r}) + j\omega\rho_m(\mathbf{r}) = \frac{I_c}{A}, \tag{6}$$

where ∇_S is the surface divergence, I_c is the coupling current, A is the area of the connection patch on the EM object. The right hand side of Eqn. (6) contributes to the electric field of both the exterior medium and the interior medium as an extra electric charge density by Eqns. (2a and 3). Also the coupling from the EM object to the circuit is obtained by enforcing the exterior medium electric potential on the connection patch to be equal to the voltage of the corresponding circuit node. Thus we obtain a complete set of linear equations that can be solved as a coupled system to obtain the EM currents as well as the circuit variables. Also we can obtain a terminal model for the EM object that can be used directly as a stamp for the MNA matrix to solve for different circuits with the same EM object. The electric field can be found inside the EM object by plugging in the EM unknowns in the interior medium problem

$$\mathbf{E}_i^{tot} = -j\omega\mathbf{A}_i - \nabla\phi_i - \frac{1}{\varepsilon_i}\nabla \times \mathbf{F}_i + \mathbf{E}_i^{exc} \tag{7}$$

The true electric current in the interior of the EM object can be found from the field as

$$\mathbf{J}_i^{True} = \sigma\,\mathbf{E}_i^{tot} \tag{8}$$

This is also important to note that the formulation is sufficiently robust to incorporate the effect of incident field, both interior and exterior to the matrix in addition to the circuit excitation.

Results

Resistance and inductance of a rectangular cross section interconnect is extracted by observing the terminal voltage difference across the object with a given current excitation. Fig. 2(a,b) depicts the frequency behaviour of the inductance and resistance of a Copper trace of dimension $5mm \times 0.5mm \times 0.5mm$. In Fig.2a the inductance curve shows the expected leveling off at low- and high frequencies, and also shows excellent agreement with the DC inductance obtained from a volumetric solver, and the AC inductance obtained from a surface-PEC solver respectively. Fig. 2b show the excellent agreement of the quasi-static resistance obtained using the presented method, with the analytic resistance computed from the skin-depth thickness at a given frequency. Fig. 3 shows the effect of radiation resistance captured through a full-wave simulation. Fig. 4 depicts the volumetric current flow inside the conductor by using Eqns.(7,8). As the skin depth decreases with increase in the conductivity, the current flow becomes dominant near the surface and the corners, and the ratio between the maximum and minimum current density increases. Also, the computed logarithmic slope of the current density is plotted in Fig. 4, against the expected skin-depth based slope along the diameter of a Copper cylinder of diameter 1mm. and length 5 mm.

Conclusions

A seamless broadband method is presented to model the EM behaviour of conducting structures in microelectronic circuits through a unified matrix. The EM modeling as well as the coupling scheme has been explained. Results have been presented to validate the proposed broadband technique against existing commercial solver. Further post-processing is performed in order to determine the volumetric current flow inside the conductor using a surface only formulation, and comparisons to skin effect models is also demonstrated. Continuing work includes low-frequency stabilization and fast multilevel solvers based on QR methods.

Acknowledgements

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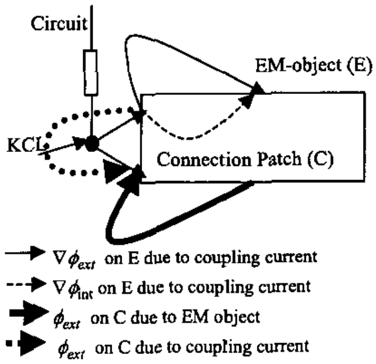

Fig. 1 Circuit coupling

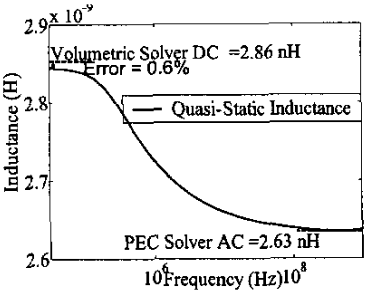

Fig. 2a Frequency dependence of inductance

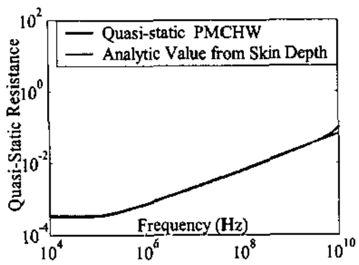

Fig. 2b Frequency dependence of resistance

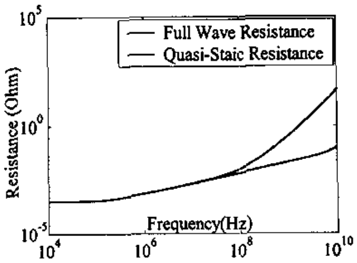

Fig.3 Full-wave radiation resistance

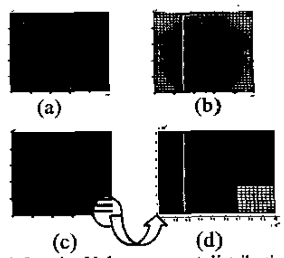

Fig 4. Interior Volume current distribution

(a) $\sigma = 5.8 \times 10^4$, J_{max}/J_{min}=99%

(b) $\sigma = 5.8 \times 10^6$, J_{max}/J_{min}=10.3%

(c) $\sigma = 5.8 \times 10^7$, J_{max}/J_{min}=0.09%

(d) Zoomed view of a corner of (c)

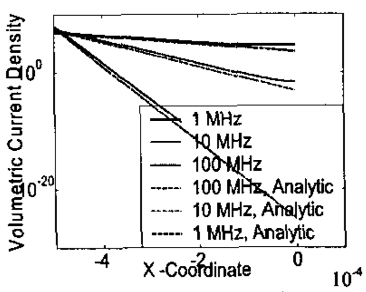

Fig 5. Skin-depth decay of current in volume